Generation and Conversion of Transverse Gaussian Laser Modes

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Introduction

We built a two-stage Mach-Zehnder interferometer to study the intensity and phase structure of open-cavity laser Hermite-Gaussian (HG) modes converted to helical Laguerre-Gaussian (LG) and sinusoidal LG modes. The helical variation are of interest due to their orbital angular momentum. Modes of the sinusoidal form are of interest because of their applications in gravitational wave astronomy.

Solutions to the Paraxial Wave Equation

The Helmholtz equation describes the behavior of waves and can be simplified with the approximation

$$sin\theta \approx \theta$$

The resulting paraxial wave equation has solutions of the Hermite polynomials multiplied by Gaussian functions. These form a complete basis set of traveling waves and comprise the familiar TEM_{jk} modes (fig. 1).

$$E_{nm}(x,y,z) = E_0 \frac{w_0}{w(z)} H_n\left(\sqrt{2}\frac{x}{w(z)}\right)\exp\left(-\frac{x^2}{w(z)^2}\right)\cdot H_m\left(\sqrt{2}\frac{y}{w(z)}\right)\exp\left(-\frac{y^2}{w(z)^2}\right) \exp\left(-i\left[kz-(1+n+m)\arctan\frac{z}{z_R}+\frac{k(x^2+y^2)}{2R(z)}\right]\right) \quad [1]$$

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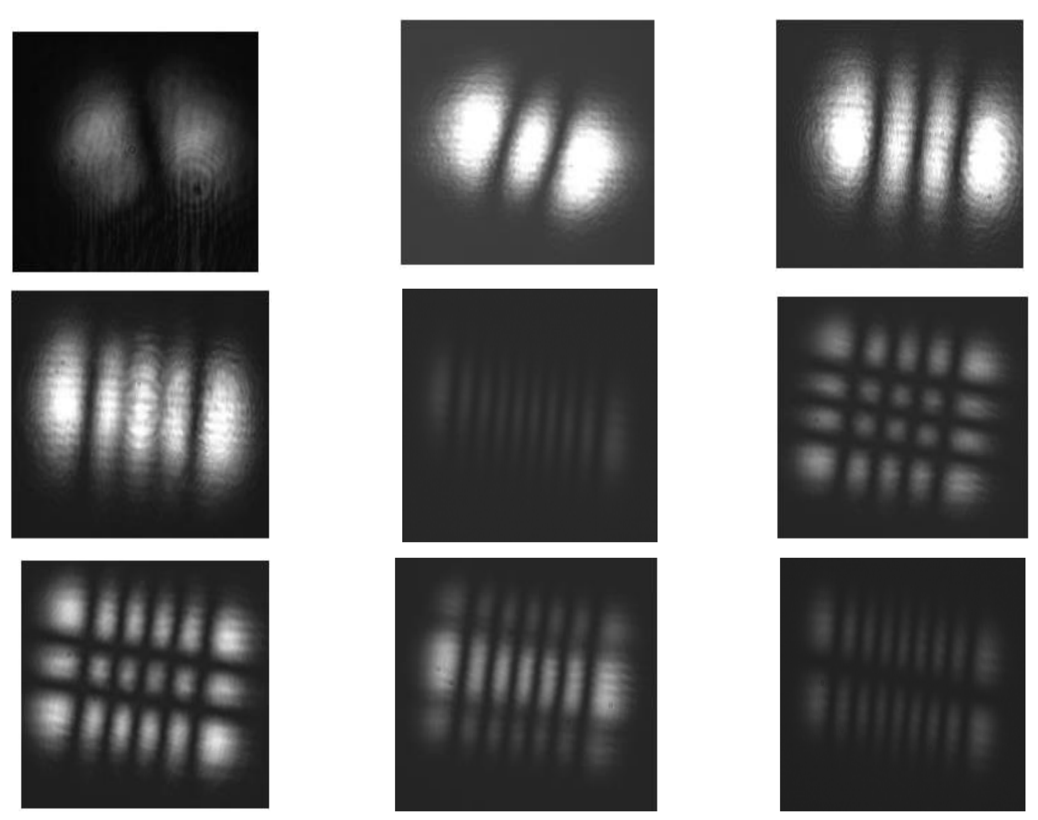


Fig. 1: HG_0^1 , HG_0^2 , HG_0^3, HG_0^4, HG_0^9 , HG_4^3 , HG_5^2 , HG_6^2, HG_8^1 modes

In cylindrical coordinates, the helical LG functions, Laguerre polynomials multiplied by Gaussian functions, are also solutions. LG modes feature circular symmetry and orbital angular momentum about their helical wavefront (fig. 2).

$$E^{hel}_{p,l}(r,\phi,z) = \frac{E_0}{w(z)} L_p^{(|l|)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|}\left(\frac{2r^2}{w(z)^2}\right) \times \sqrt{\frac{2p!}{\pi(|l|+p)!}}\; e^{-ik\frac{r^2}{2q(z)}+il\phi}\; e^{i(2p+|l|+1)\Psi(z)} \quad [1]$$

The combination of two oppositely handed helical LG modes forms a third basis set of solutions .These sinusoidal LG modes no longer have orbital angular momentum.

$$E^{sin}_{p,l}(r,\phi,z) = \frac{E_0}{w(z)} L_p^{(|l|)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|}\left(\frac{2r^2}{w(z)^2}\right)\sqrt{\frac{2p!}{1+\delta_{0l}\pi(|l|+p)!}} \times e^{-ik\frac{r^2}{2q(z)}}\; e^{i(2p+|l|+1)\Psi(z)}\begin{Bmatrix}\sin(l\phi)\\ \cos(l\phi)\end{Bmatrix} \quad [1]$$

Orbital Angular Momentum

Orbital angular momentum (OAM) is separate from the intrinsic spin angular momentum (circular polarization) of light. The wavefront of the helical LG mode demonstrates this OAM (fig. 2).

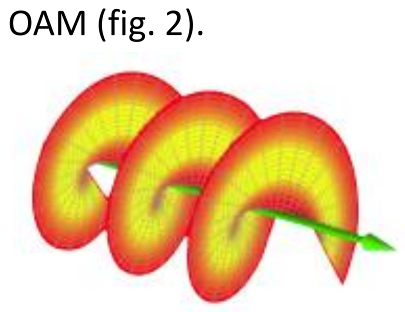
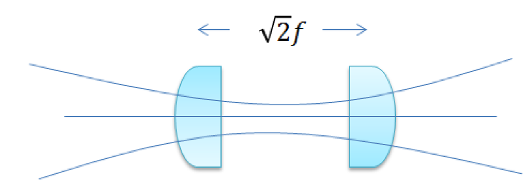


Fig. 2: Helical wavefront

Open-Cavity HeNe Laser and the Astigmatic Mode Converter

We used an open cavity, 632nm HeNe laser to generate HG laser modes. Use of an aperture within the cavity allowed us to cut off higher angle reflections that would generate very high order modes. A 10μm wire was translated transverse to the beam in order to promote modes that had gaps in their E-fields at the position of the wire. This enabled the selection of a single higher order mode.

The astigmatic mode converter works by manipulation of the Gouy phase acquired during the propagation of the beam through its waist. [2]



$$\varphi_G(z) = -arctan\left(\frac{z}{z_R}\right)$$

The first cylindrical lens introduces an astigmatism and definite phase difference ($\boldsymbol{\pi}/2$) between HG components oriented along axes of astigmatism. The second cylindrical lens removes the astigmatism, leaving a modal beam which has a constant intensity distribution throughout propagation.

Generation of Laguerre- and Sinusoidal-Gaussians

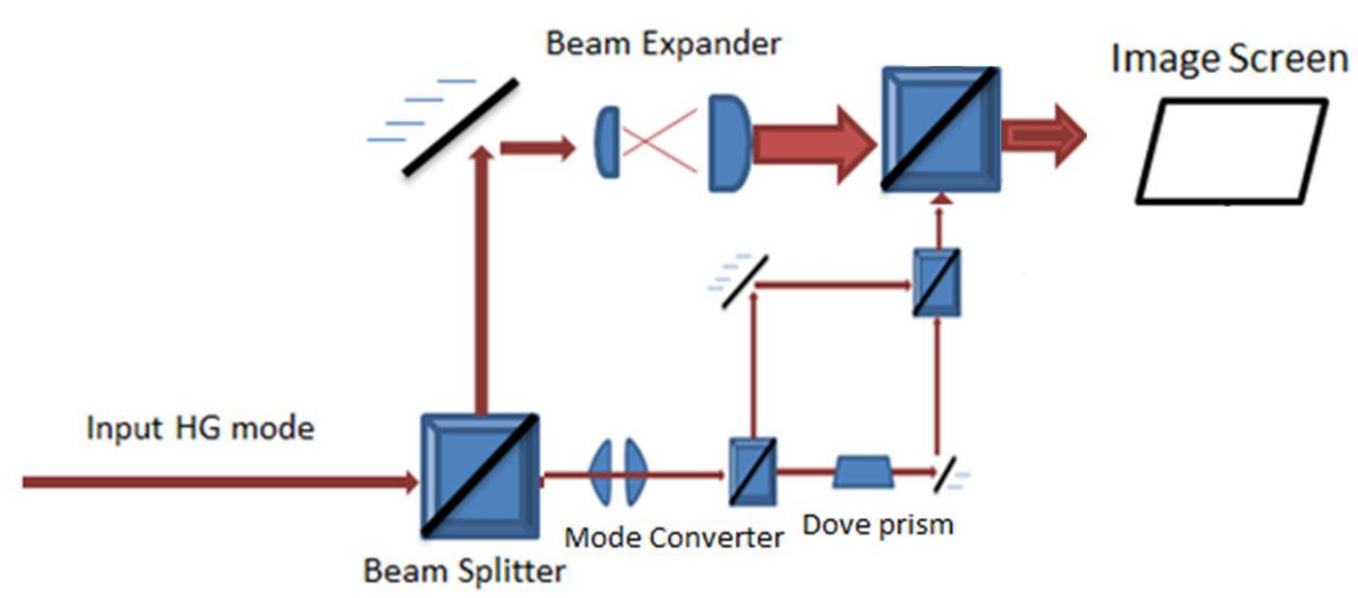


Fig. 3: Compound Mach-Zehnder setup

A compound Mach-Zehnder interferometer was used for generation of the LG based modes (fig. 3).

Outer Interferometer:
- An HG mode is either expanded into a reference plane wave (upper path) or is converted (fig. 4) (inner interferometer). [3]

Inner Interferometer:
- The helical LG mode is transmitted (upper path) and the Dove prism reverses the handedness of the same mode (lower path). These recombine to form a sinusoidal LG mode (fig. 6).

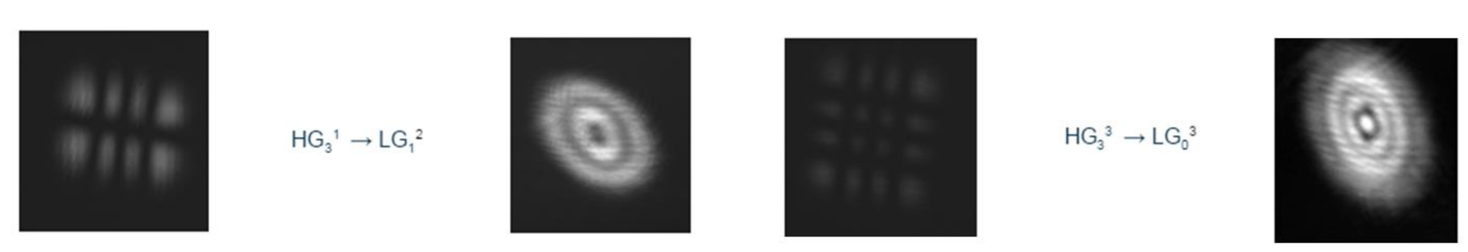


Fig. 4: HG to LG_{hel} conversions

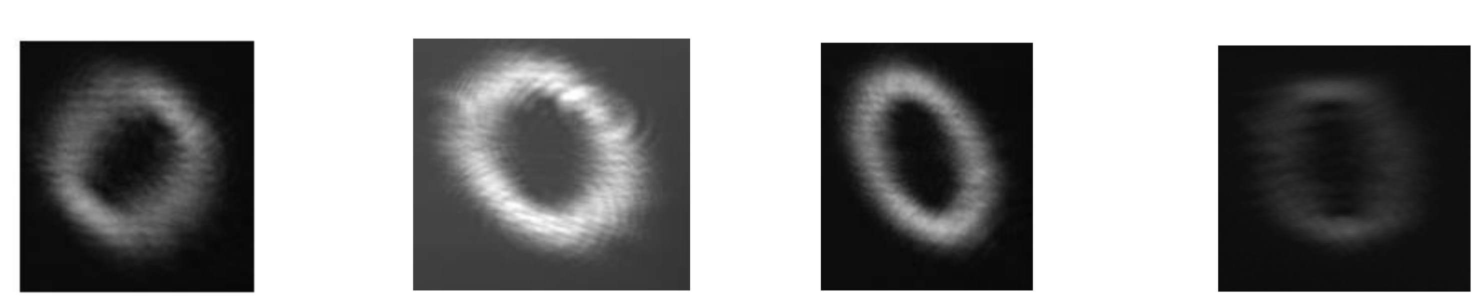


Fig. 5: Arbitrary high order LG modes: LG_5^0 , LG_6^0 , LG_7^0 , LG_8^0

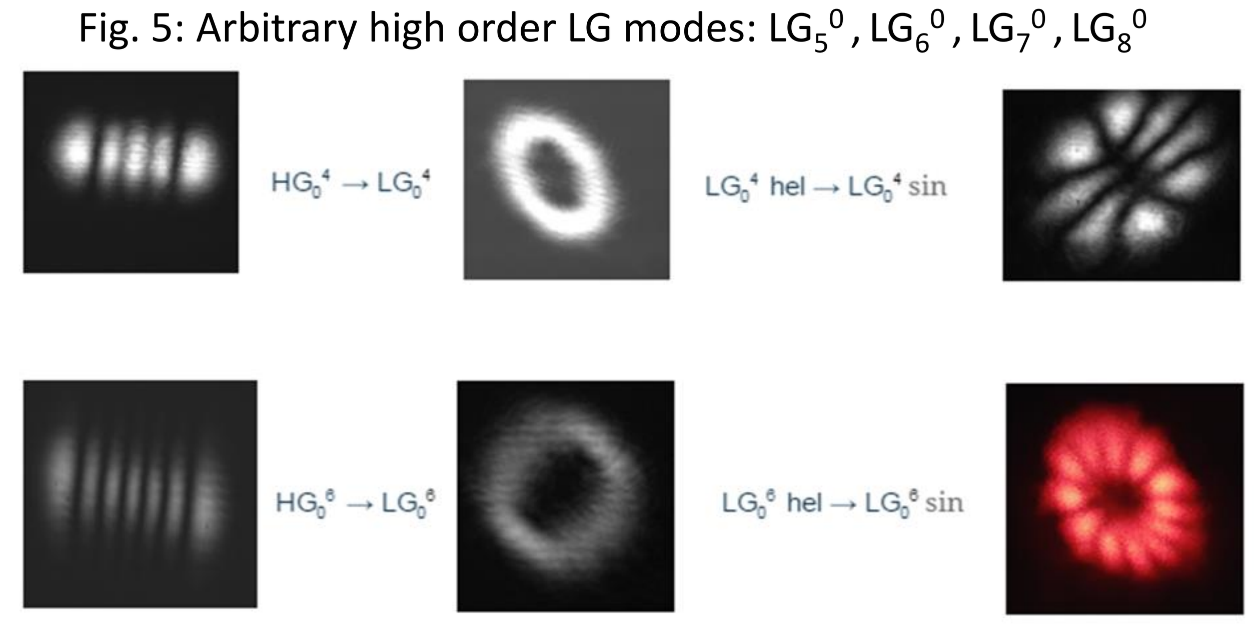


Fig. 6: Full HG to LG_{hel} to LG_{sin} conversion

Phase Analysis of LG Modes

With an expanded HG lobe as a reference, either path in the inner interferometer could be blocked to view either handed helical LG mode.

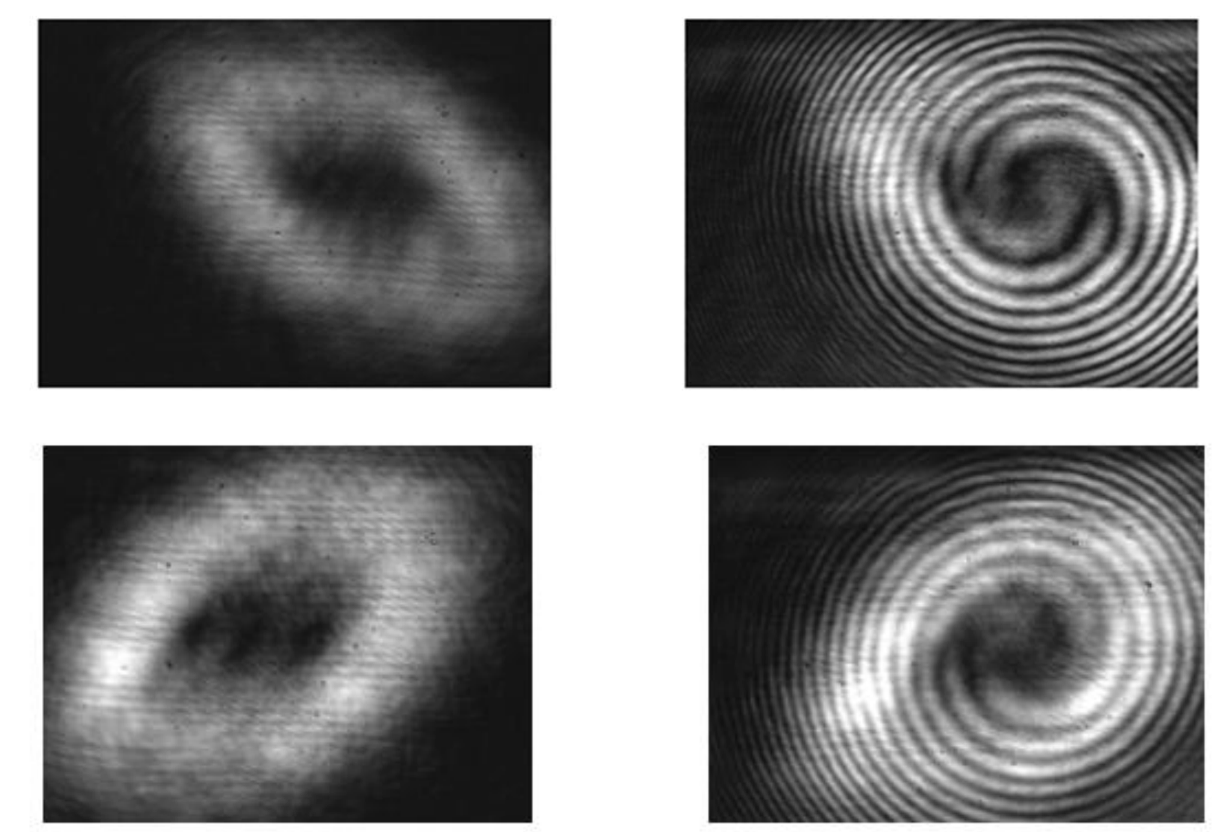


Fig. 7: Oppositely handed helical LG modes with their resultant phase patterns. Counter clockwise and clockwise spirals respectively. The number of spirals corresponds to the order of the mode.

Without blocking either inner path, a sinusoidal LG mode is sent through to interfere with the reference plane wave.

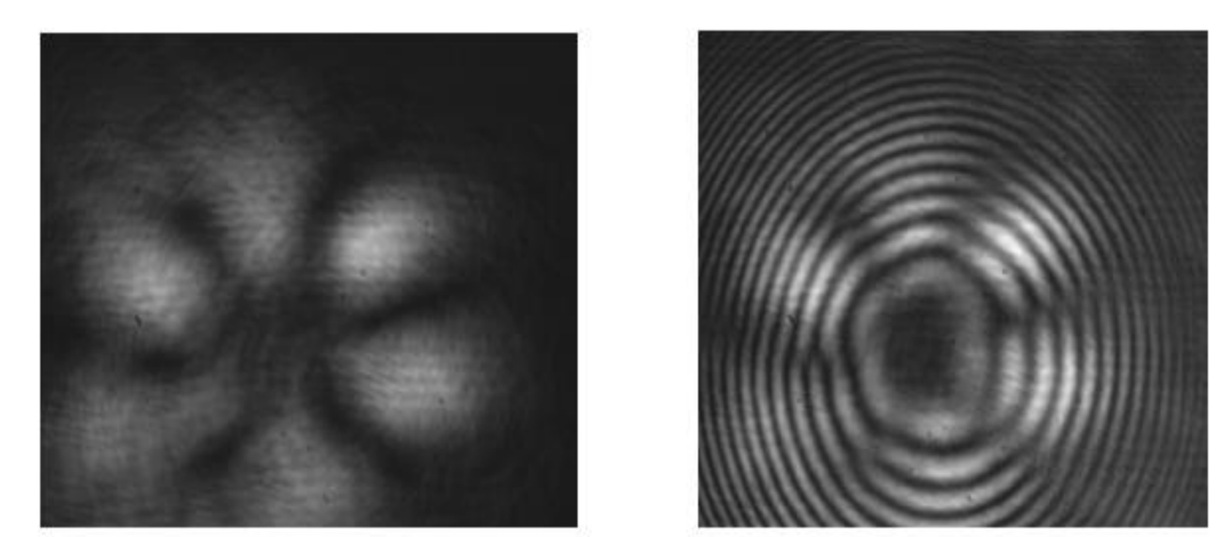


Fig. 8: A sinusoidal LG_0^3 mode shows a striated petal pattern and an offset in the first three interference rings.

LG modes can provide a wider intensity distribution across mirrors in interferometers used for the study and detection of gravitational waves. The larger, and more uniform distribution of intensity in these higher order LG modes averages better over the surface of the mirror, reducing thermal-aberration induced distortions. LG modes are compatible with the spherical mirrors common to these interferometers. [4]

Acknowledgments and References

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