Diffracton by Two Non-Coplanar Cylinders

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We have diffracted 632.8 nm laser light using two 635 μm-diameter steel cylinders to create the Babinet principle inverse of two slits. One rod is then translated along the direction of beam propagation. We have observed a changing spatial frequency on each side of the central maximum which is proportional to the fringe order, and that changes predictably with offset. Our simulation takes a Green's function approach to the Rayleigh-Sommerfeld integral and supports the measurements. A corresponding equation is derived from first principles.

1. Introduction

In typical double-slit or double-bar diffraction experiments, it is usually assumed that the incoming light propagates perpendicular to the plane of the diffracting objects. Additionally, the objects are assumed to be flat such that they have no projection along the beam. With these conditions satisfied, it is a textbook problem to construct the corresponding aperture functions and predict the form of the resulting far-field diffraction pattern. In this experiment, cylinders are used rather than flat obstacles, and one cylinder is translated along the beam direction so that the two cylinders are no longer in the same plane perpendicular to the beam path. This is significantly more complicated to simulate and is a relatively unexplored topic. Our experiment seeks to answer two questions. First, what is the difference between the far-field pattern of a flat blockage and that of a cylinder? Babinet's principle allows comparison of a double-obstacle to a double-aperture pattern [reference], but does this break down with 3-dimensional obstacles? It has been shown that a cylinder is distinguishable from a flat blockage of the same diameter in the near field. [reference] We examine how this translates to the far-field. Second, if one cylinder is translated along the direction of the beam, what is the resulting effect on the Fraunhofer diffraction pattern? When one rod is translated so that the incoming light does not hit both simultaneously, a spatial Fourier transform of the light field is insufficient; light has already begun diffracting from the first cylinder before it hits the second one. To be thorough, it is necessary to take a close look at the Rayleigh-Sommerfeld integrals to understand the near-field diffraction occurring between the two cylinders. We may propagate the near-field diffraction past both cylinders and find the 2-D electric field, take its Fourier transform, and observe a precise prediction of the diffracted intensity.

2. Equation and Simulation

The textbook double-slit or double-bar far-field diffraction pattern is a series of interference fringes, with spacing at the viewing screen:

$$F = \frac{z\lambda}{d} \quad (1)$$

where F is the fringe spacing, z is the distance between the slits and the viewing screen, λ is the wavelength of the incident light, and d is the center-to-center separation of the slits.

These fringes are modulated by a sinc^2 function. The end result is a series of fringe packets, with spacing

$$F_{env} = \frac{z\lambda}{a} \quad (2)$$

where a is the slit width.

2.A. Derivation of Algebraic Equation

It is possible to derive a single expression for the interference fringe spacing when one cylinder is translated from first principles and some simple geometry.

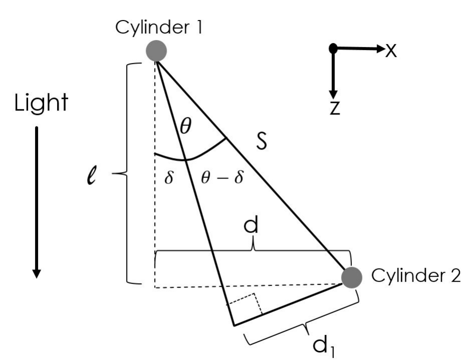

Fig. 1. Diagram of Light Hitting Cylinders

Light incident on the first cylinder diffracts at some angle to the direction of propagation, δ, and hits the offset cylinder.

$$d_1 = S\sin(\Theta - \delta) \quad (3)$$

Recall the condition for a double slit envelope maximum,

$$\sin(\delta) = m\lambda \quad (4)$$

where m is the envelope diffraction order. We may use this, and the condition that δ is sufficiently small, to expand Eq. 3:

$$d_1 = d - \frac{\ell m\lambda}{a} \quad (5)$$

From this, we determine the fringe spacing F' when there is some offset ℓ.

$$F' = \frac{z\lambda}{d_1} = \frac{z\lambda}{d}\left(\frac{1}{1 - \frac{\ell m\lambda}{da}}\right) \approx \frac{z\lambda}{d}\left(1 + \frac{\ell m\lambda}{da}\right) \quad (6)$$

Now we can solve for the difference between the fringe spacing in the coplanar case (F) and the fringe spacing in the offset case (F'). Some simple algebra yields

$$\Delta F = F' - F = \left(\frac{\lambda}{d}\right)^2 \frac{z}{a} m\ell \quad (7)$$

From the well-known double slit equations,

$$p = \frac{md}{a} \quad (8)$$

where p is an interference fringe order and m is an envelope order. Finally,

$$\left(\frac{\Delta F}{\ell}\right)_{avg} = \frac{z\lambda^2}{d^3}\left(\frac{p+q}{2}\right) \quad (9)$$

p and q are both interference fringe orders; these determine the range of fringes used to determine spacing. As the data will show, the choice of p and q does matter; the fringe spacing in the non-coplanar case does not remain constant.

2.B. Simulation

We perform a numerical simulation as well; because the incoming Gaussian beam does not hit the two cylinders simultaneously, a simple Fourier transform is insufficient. When cylinder 2 (see Fig. 1) is translated, the light diffracts around the first cylinder, propagates through free space, and then diffracts a second time around the final cylinder. To model this, we use the Rayleigh-Sommerfeld integrals to find the light field intensity at a number of X-Y planes (see Fig. 1) up until the light passes the second cylinder. We then take the Fourier transform of the light field in the final plane-just past the second cylinder- to obtain a simulated far-field intensity pattern.

[graphic?]

3. Experimental Procedure

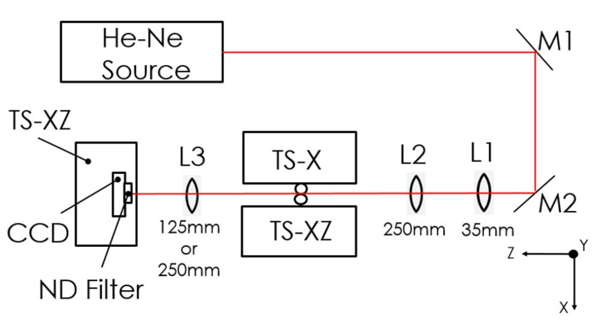

Fig. 2. Experimental Setup

We use two 23-gauge pin nails (635 μm in diameter) as our cylinders and a 632.8 nm He-Ne laser as our light source. Each cylinder is placed on a translational stage; cylinder 1 translates perpendicular to the beam only (along x-axis), and cylinder 2 translates both perpendicular and parallel (along x and z axes). This allows for easy spacing and offset adjustment.

The beam is expanded to 6.4 mm in diameter at the $\frac{1}{e^2}$ point. The 250mm lens shown in figure 2 acts as a Fourier lens that produces a far-field pattern at a small distance. As a result we may substitute the z in Eq.9 for f = 250 mm.$^{[reference]}$ This lens may also be exchanged for a 125 mm lens that allows for the cylinders to be directly imaged onto the camera (see Fig. 2). This enables precise measurement of their widths and x-axis separation. We use a DataRay $\frac{1}{2}$" WinCamD CCD camera for imaging and data collection. This camera has 9.3 μm pixels and 14 bit depth resolution. No precise orientation is set for the polarization of the incoming light, as we have determined visually that polarization has little to no effect on the observed intensity profiles. $^{[reference]}$

The cylinders are observed at a coplanar position, and with cylinder 2 at various positive and negative displacements.

4. Results and Analysis

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5. Conclusion

6. Acknowledgements

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References