July 8, 2008

Today Dr. Metcalf continued his presentation on Matrices and Quantum Mechanics. We started by defining $\Omega$ as a complex number representing the interaction between an atom and anything else. Using some algebra, we were then able to rewrite the Hamiltonian as:

$$\begin{pmatrix}
E & \hbar \Omega \\
\hbar \Omega^* & -E
\end{pmatrix}$$

By including $\Omega$ in the Hamiltonian, we made it a general operator for any type of interaction involving our two-state system since the value of $\Omega$ is dependant on the interaction. For example, if nothing is interacting with the atom then $\Omega$ is zero, and the Hamiltonian will return values of either $E$ or $-E$ depending on whether the atom is in the ground state or the excited state.

We can go a step further and use the general Hamiltonian that we just found to determine the general eigenvectors for any two-state system. Once all the algebra has been done out, we find that:

$$\lambda = \pm \sqrt{E^2 + \hbar^2|\Omega|^2}$$

Using this, we can then find the Eigenvectors of the general Hamiltonian operator. Once the brunt of the algebra has been completed, we end up with:

$$a^2 = \frac{\hbar^2|\Omega|^2}{E^2 - 2E\lambda + \lambda^2 + \hbar^2|\Omega|^2}$$

Which we can rewrite as:

$$a^2 = \frac{y^2}{E^2 - 2E\lambda + \lambda^2 + \hbar^2|\Omega|^2}$$

$$a^2 = \frac{y^2}{x^2 + y^2}$$
Because \( a^2 + b^2 = 1 \):

\[
b^2 = \frac{x^2}{x^2 + y^2}
\]

\( a^2 + b^2 = 1 \) also describes a unit circle however, and we can use the circle to rewrite \( a \) as \( \sin \theta \) and \( b \) as \( \cos \theta \), which yields an eigenvector of:

\[
\begin{pmatrix}
\sin \theta \\
\cos \theta
\end{pmatrix}
\]

What makes this eigenvector interesting is the fact that for certain values of theta, the system will occupy both states at the same time. When one observes the system, it will choose either one state or the other, however, it must be thought of as being in both states at the same time when it is not being observed. This is one of the most important differences between Quantum Mechanics and Classical mechanics.

\[\text{—NityanNair}\]