A Complete Ray-Optic Analysis of the Mirage® Toy

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The Mirage® (Opti-Gone International) is a well-known commercial demonstration (PIRA index number 6A20.35) that uses two opposed concave mirrors to project a real image of a small object into space. We have studied the image formation in the Mirage® by both standard matrix methods in the paraxial approximation and by exact ray tracing with the program BEAM2 (Stellar Software), with particular attention to additional real images that can be observed when the mirror separation is uniformly increased. We find that the three readily observed secondary images correspond to 4, 6, or 8 reflections, respectively, contrary to previous reports.
keywords: optics demonstrations, real image, ray tracing ??

1. Introduction

The Mirage® [1] is a well known optics demonstration [2] that is very popular in our laboratory and at open house events. It consists of two horizontal concave mirrors that work together to project a real image of a small object placed on the lower mirror through an aperture in the upper one. Visitors are fascinated by the realistic image that "floats in space" and are challenged and involved when they are asked what will happen when it is viewed through a magnifying glass or mirror, or illuminated by a laser beam. An often overlooked [little-noticed or rarely noticed] feature of the Mirage® is the appearance of additional real images when the separation between the two mirrors is uniformly increased by carefully raising the upper mirror. These secondary images occur at additional distances [mirror separations] of 3.1, 4.5, and 5.3 cm and are alternately inverted or not inverted compared to the primary image.

We have studied image formation in the Mirage® toy by both standard 2x2 matrix ray-optics methods in *Mathematica*[3] and by exact ray tracing with BEAM2[4]. The problem provides a very good introduction to geometrical optics and to these two very useful and pedagogically valuable software tools. The project is typical of those carried out at the Laser Teaching Center, as discussed elsewhere at this conference[5, 6].

2. Description

The Mirage® is readily available from various suppliers of science education products at a cost of about US $35 plus shipping. Its two halves (upper and lower) are made from a durable black plastic [what?] by injection molding. The mirror surfaces appear to be optically quite accurate except for an irregular portion ~2 mm in diameter at the center of the lower mirror. This imperfection is of no consequence as it is normally covered by the object placed there. The optical surfaces are protected from tarnishing by a durable over-coating, but are quite sensitive to damage from fingerprints or scratches. We have found that cleaning is best done by wetting the surface with household detergent and flushing with a copious stream of hot water; residual water droplets can be blotted dry with a paper towel, but the surface must never be stroked or rubbed. A very similar device marketed by a German company and sold at a somewhat lower price is of much lower optical quality (obvious irregularities across its surface) and should be avoided.

The description of the Mirage® on its packaging and elsewhere is misleading in several ways. The trade-marked name of the device has of course no relation to the refraction phenomenon by that name, and the term "3-D reflection hologram" is also misleading. The packaging further states that "optical surfaces are crafted to 5/1,000,000,000 ths of an inch." This figure corresponds to roughly one Ångstrom (10^{-10} meters) and was meant to be indicative of the extreme thinness of the aluminum reflective coating[7].

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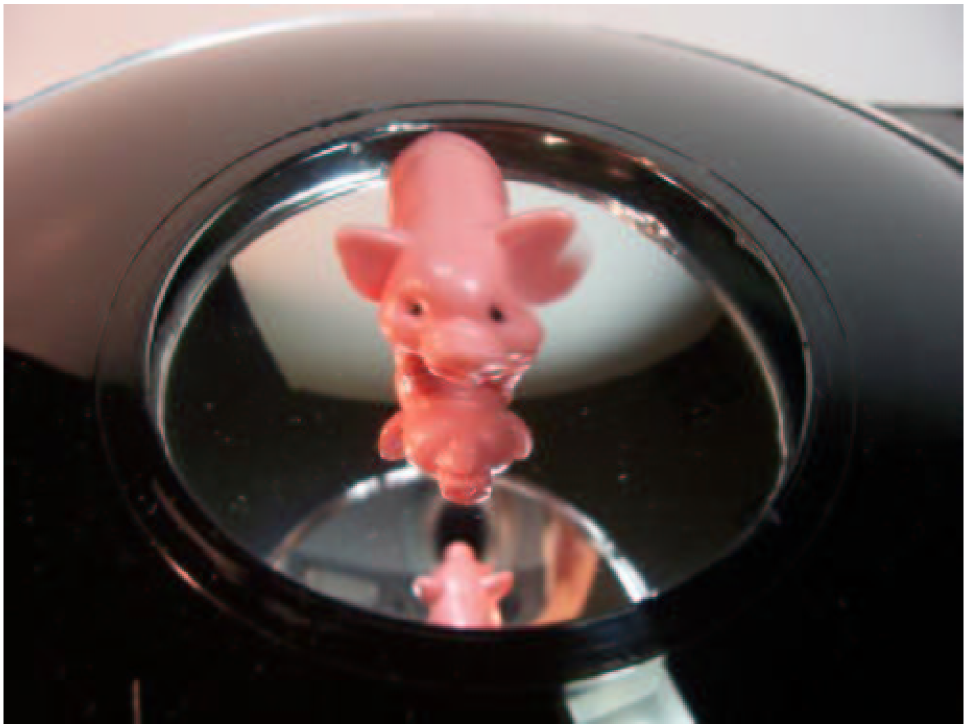

Figure 1: Photograph of the Opti-Gone Mirage$^{®}$ in our laboratory showing, from top to bottom, the projected real image, the projected reflection, and the actual object. The two images appear larger than the actual object primarily because the camera is close to the device. [Photo courtesy Azure Hansen.]

3. History

The Mirage$^{®}$ has an interesting history[7] that begins with a chance observation by a custodial worker, Caliste Landry, in the physics department at the University of California, Santa Barbara some four decades ago. One day Landry happened to be cleaning a stack of large World War II surplus searchlight mirrors that had been stored away in a closet. (Such mirrors have a central aperture for the arc lamp support.) Landry was startled and fascinated to see a realistic illusion of "dust that couldn't be cleaned." He reported his observation to a young faculty member in the department, Virgil Elings[8], who quickly recognized the optical principles involved and the novelty and potential utility of devices of this type [reword].

Elings and Landry filed for a patent for an "Optical Display Device" in 1970, and it was granted two years later[10]. The patent is interesting for what it does and does not say about the optics of the device. For example, figures 3 and 4 in the patent show the second solution for two reflections and the non-inverted secondary image after four reflections, respectively. (See discussion in Sections 5 and 6 below). The accuracy of the images is not discussed, and the drawings show only a symmetrical pair of rays. The patent also mentions without any further discussion the possibility of having one mirror not be concave, or unequal curvature. etc. [to be completed]

Michael Levin, the founder of Opti-Gone International, first became aware of the Elings device when he came across an expensive glass version of it (made by Elings and his son [Mike or Jeff?]) at a San Francisco gift shop. Levin had prior experience with commercial ventures related to optics, having been involved with the Laserium[11] shows in the 1970's, and could see the potential of marketing a more affordable version to a wider audience. By 1977 he had founded Opti-Gone International, whose sole products remain the standard (Model 2000) Mirage$^{®}$ and a much larger version (Model 22) used for dramatic displays at museums and the like. [When did he acquire the rights to the Elings patent?].

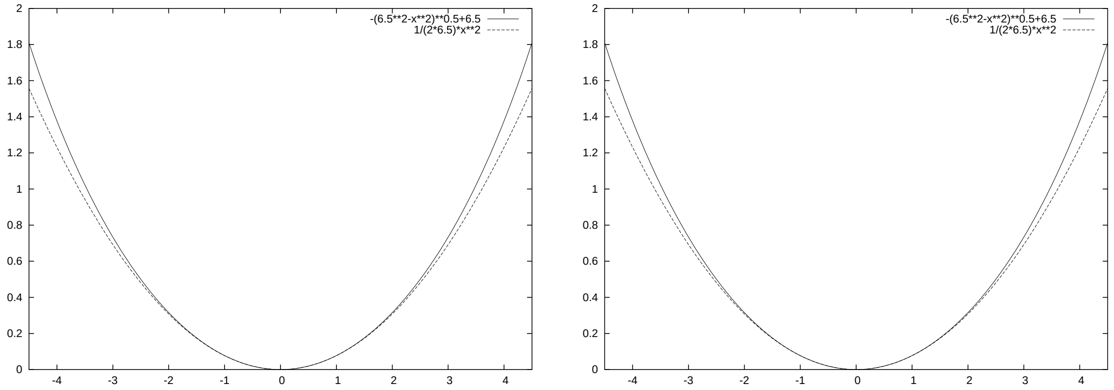

Figure 2: Results of the shape analysis ...

4. Geometry

Although the Mirage® is often described as having parabolic surfaces, we decided to study its geometry with no prior assumptions other than that the mirrors are surfaces of revolution. We first used a caliper and steel ruler to measure the diameter of the lower mirror c and the perpendicular distance (sagitta) from the midpoint of that chord to the mirror surface h, taking care to account for the lip at the mirror's edge. Using the relationship $R = c^2/(8h) + h/2$ we deduced a radius of curvature $R = 7.09$ inches. A circular template of this radius cut out from paper did not match the surface, so it was apparent that this was not spherical. A smaller circular template (R=6.35 inches) did match the surface well near its center, however. Finally, a parabolic template with the same curvature at the vertex was constructed. It matched the surface well

Measurements confirmed that the vertices of the two mirrors in the Mirage® are separated by one focal length (8.06 cm) as expected. The sagitta method was used to account for the aperture of the upper mirror, which has a diameter of ?? cm.

5. Matrix Analysis

The matrix optics technique for ray tracing is well known[12, 13]. The (radius, angle) form we used applies to spherical optical elements on a common axis, where the angle θ between rays and that axis is sufficiently small that the paraxial approximation $\tan(\theta) \simeq \sin(\theta) \simeq \theta$ applies.

The matrix analysis had two parts. First we studied the magnification and displacement of the primary image as a function of the displacement of the object above the lower mirror. Then we studied the secondary images by finding all mirror separations d/f at which an image forms at the upper surface, after 2, 4, 6, 8, or 10 reflections.

For n reflections there are n solutions, but many of these are for mirror separations less than the focal length, or at relatively large separations. The first solution for 2 reflections ($d = f$) is what is normally observed; a second solution occurs at $d = 3f$. This second solution has been discussed before by other authors, but they erroneously associated it with one of the visible secondary images.

condition on matrix element.

$$\text{Drift } \mathbf{D} = \begin{pmatrix} 1 & d-x \\ 0 & 1 \end{pmatrix}$$

$$\text{Reflection } \mathbf{R} = \begin{pmatrix} 1 & d-x \\ 0 & 1 \end{pmatrix}$$

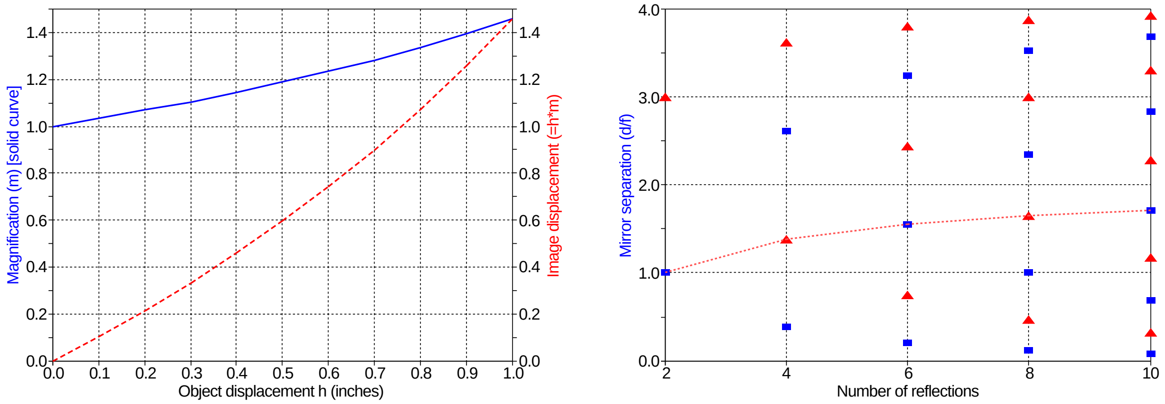

Figure 3: Results of the matrix analysis. The predicted secondary images are either inverted (triangles) or not (squares). The dashed line connects the sequence of images observed when the top mirror is lifted.

Product matrix $\mathbf{P} = \mathbf{DRDRD}$

The results of the matrix analysis are summarized in the Figure. The number of secondary images is equal to the number of reflections, and half of each series of images is inverted. The dashed line connects the images observed when the top mirror is lifted.

6. Exact Ray Trace Analysis

The BEAM2 software [4] proved to be an excellent tool for the exact ray trace studies, limited yet quite adequate. (It only handles surfaces of revolution, but these can be formed by an arbitrary conic section.) It was quite easy to learn, especially with the detailed examples published by Atneosen and Feinberg[14]. Finally, it is quite affordable, just US $89 for a single user copy, the same price quoted in the 1991 paper[14].

We started by confirming that the primary image formed by parabolic mirrors one focal length apart is perfect, with no geometrical aberrations. This result is expected ...

How to use it, ray and optic tables, cite.

Primary image - parabolic vs. spherical

Secondary images - not perfect even with parabola.

We have only begun to explore the possible studies

7. Discussion

In reality light rays in the Mirage® are far from paraxial, and the reflecting surfaces are not spherical.

The exact ray trace reveals that many of the matrix solutions for the secondary images are strongly aberrated, or do not exist[15]. There is no aberration in the primary image, however, since the intermediate rays in this case are parallel to the optic axis, as in a telescope.

As suggested in the patent there are many more possibilities that can be explored by the methods described here, including having mirrors of different shapes and unequal curvature (focal length). This device is truly a rich playground for exploring geometrical optics.

Like all good LTC projects, the more we do the more there is to learn and explore. Confocal cavity designs. A pair of solid mirrors could be used

Acknowledgements

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References

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[1] "Mirage" is a registered trademark of its manufacturer, Opti-Gone International, Ojai, CA 93023 USA. `http://www.optigone.com`.

[2] Listed on the Pira (Physics Instructional Resource Association) database of physics demonstrations as 6A20.35. See for example,
`http://www.physics.brown.edu/physics/demopages/Demo/optics/demo/6a2035.htm`.

[3] Wolfram Research, Champaign, IL 61820-7237 USA. `http://www.wolfram.com`.

[4] BEAM2 program from Stellar Software, P.O. BOX 10183, Berkeley, CA 94709 USA, $89 plus shipping. `http://www.stellarsoftware.com`

[5] J.W. Noé, "The Laser Teaching Center at Stony Brook University," this conference.

[6] J.W. Noé, "Simple, creative projects from the optics teaching laboratory," this conference.

[7] Michael Levin, Opti-Gone International, private communication. May 2007.

[8] Elings went on to found Digital Instruments, Inc, the company which pioneered scanning probe microscopy (now part of Veeco, Inc.). There is a biography of Elings at

[9] `http://web.mit.edu/physics/alumniandfriends/profiles/elings.html`

[10] Virgil B. Elings and Caliste J. Landry, US Patent #3647284, granted March 7, 1972, expired. `http://www.google.com/patents?id=cH4wAAAAEBAJ&dq=3647284`.

[11] Laserium shows were developed by Ivan Dryer. The first was in November, 1973 in Los Angeles. `http://en.wikipedia.org/wiki/Ivan_Dryer`

[12] A. Gerrard and J.M. Burch, *Introduction to Matrix Methods in Optics*. Dover Press, 1994.

[13] Eugene Hecht, *Optics, Third Edition*. Addison Wesley Longman, 1998.

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[15] Alex Dzierba, Indiana University. Course notes for Physics 360, Physical Optics. `http://dustbunny.physics.indiana.edu/~dzierba/P360n/notes/note4.pdf`

[16] Andrzej Sieradzan, "Teaching Geometrical Optics with the 'Optic Mirage'." The Physics Teacher, Volume ??, page 534, November 1990.

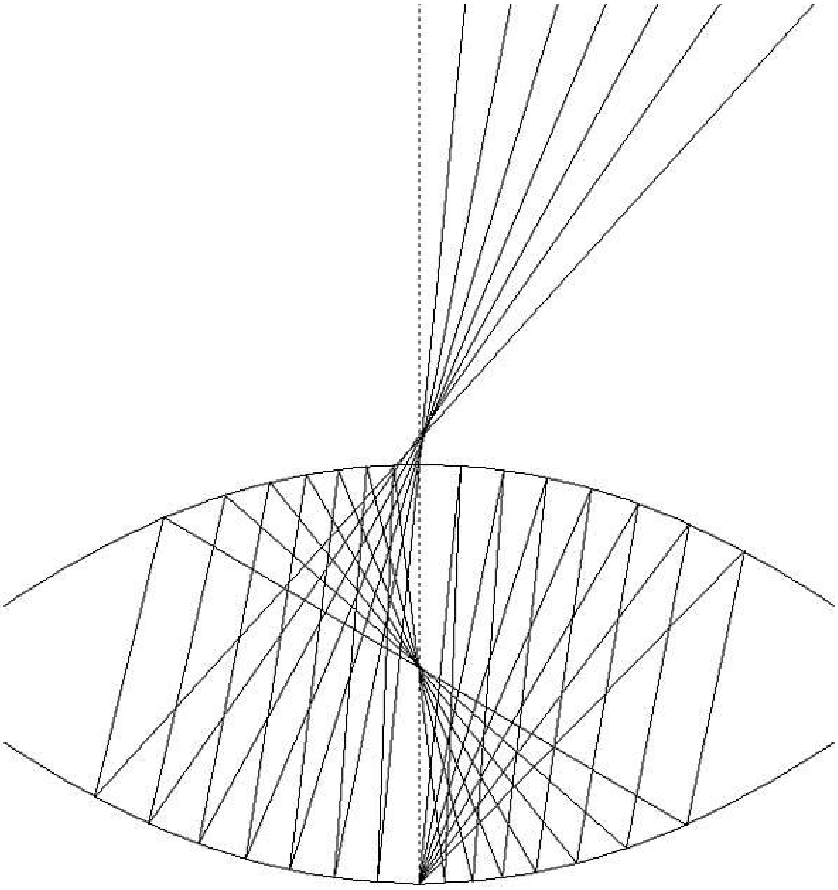

Figure 4: Ray trace for the first secondary image, with four reflections per ray.

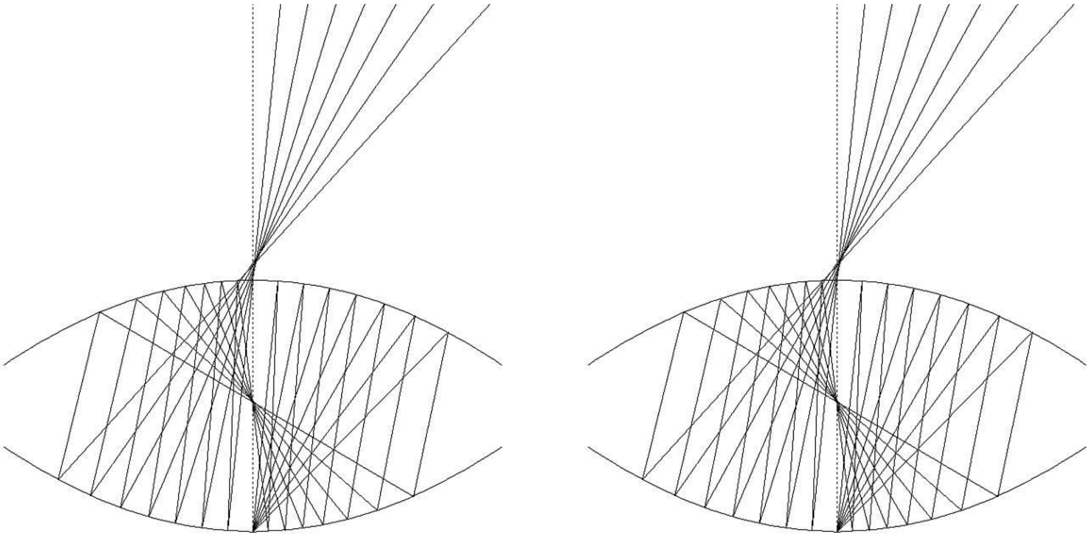

Figure 5: Ray trace for the first secondary image, with four reflections per ray.