Abstract

We present a novel technique to frequency lock a laser to an optical cavity. This technique, tilt locking, utilizes a misalignment of the laser with respect to the cavity to produce a non-resonant spatial mode. By observing the interference between the carrier and the spatial mode, a quantum noise limited frequency discriminator can be obtained. Tilt locking offers a number of potential benefits over existing locking schemes including low cost, high sensitivity and simple implementation.

The frequency locking of lasers to optical cavities is used for a wide range of scientific applications including frequency stabilization\(^1\), CW frequency conversion\(^2\), optical frequency standards\(^3\) and interferometric gravitational wave detection\(^4\). This process typically requires the generation of an error signal proportional to the difference between the laser frequency and the cavity resonance. Several methods for obtaining an error signal have been used including fringe side locking\(^5\), Hansch-Couillaud locking\(^6\), transmission locking\(^7\) and mode interference locking\(^8\). Currently the most widely used method is the Pound-Drever-Hall (PDH) technique\(^9\).

The PDH technique utilizes the beat between the carrier field and non-resonant phase modulation sidebands. The sidebands provide a reference for the phase of the carrier field reflected from the cavity. By making a measurement of the phase of the reflected field, a high sensitivity, quantum noise limited measurement of the cavity is possible. The technique presented here, tilt locking, also utilizes interference between the carrier field, and a directly reflected phase reference. In this case the phase reference is a non-resonant higher order spatial mode. This system has the same frequency response and similar sensitivity as the PDH system. Instead of electro-optic encoding and electronic decoding of frequency sidebands, tilt locking uses optical encoding and decoding of spatial modes.

A cavity decomposes a misaligned input field into a set of spatial transverse electromagnetic (TEM) modes, which can be approximated by the Hermite-Gauss functions\(^10\). Higher order Hermite-Gauss modes experience different Gouy phase shifts and thus have different resonant frequencies. Tilt locking uses the first higher order mode, the TEM\(_{01}\) mode, as the phase reference for the TEM\(_{00}\) carrier. The transverse electric field distribution for the TEM\(_{00}\) mode and a TEM\(_{01}\) mode is shown in Fig. 1(a). The error signal is proportional to the magnitude of the interference between these two spatial modes, and is given by the overlap integral\(^10\). For Hermite-Gauss modes where the entire beam is detected no interference can be measured, with the overlap integral given by,

\[
I_{0,1} = \left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{u}_{00}^*(x,y) \tilde{u}_{01}(x,y) dx dy \right| = 0
\]

where \(\tilde{u}_{00}(x,y)\) and \(\tilde{u}_{01}(x,y)\) are the electric field distributions for the normalized TEM\(_{00}\) and TEM\(_{01}\) modes respectively. The integral is zero due to the orthogonality of the Hermite-Gauss modes and no error signal is obtained.

To efficiently measure the interference between the two spatial modes we use a method similar to that used in auto-alignment systems\(^11,12\). The reflected beam is detected on a two element split photodiode as shown in Fig. 1(b) in such a way that each lobe of the TEM\(_{01}\) mode falls in a separate half of the photodiode. The error signal is given by the subtraction of the photocurrents from both diode halves with TEM\(_{00}\) on resonance and (d) slightly off resonance.
\[ I_{0,1} = \left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{u}_{00}(x, y) \tilde{u}_{01}(x, y) \, dx \, dy \right| - \left| \int_{-\infty}^{0} \int_{-\infty}^{\infty} \tilde{u}_{00}(x, y) \tilde{u}_{01}(x, y) \, dx \, dy \right| \] (2)

As the \( \tilde{u}_{00}(x, y) \tilde{u}_{01}(x, y) \) product is antisymmetric, breaking the overlap integral at the origin maximizes the error signal.

The TEM\(_{01} \) mode arises from two types of misalignment: beam displacement and beam tilt. For tilt locking, the input beam is aligned and mode matched to give only the TEM\(_{00} \) mode and a TEM\(_{01} \) tilt mode. Beam tilt produces a TEM\(_{01} \) mode which has \( \pi/2 \) phase shift in one half plane and \( -\pi/2 \) in the other, relative to the TEM\(_{00} \) mode. Figure 1(c) shows a simple electric field vector diagram of the TEM\(_{00} \) and TEM\(_{01} \) tilt modes on the two halves of the photodiode when there is no phase shift added by the cavity. This occurs when the TEM\(_{00} \) mode is exactly resonant with the cavity and the TEM\(_{01} \) is non-resonant. On the left half the TEM\(_{01} \) adds to the TEM\(_{00} \) mode with \( \pi/2 \) phase while on the right half, the TEM\(_{01} \) adds with \( -\pi/2 \) phase. The power, proportional to the square of the resultant vector, is equal on each half. Thus the error signal, obtained by subtracting the photocurrents from the two halves of the photodiode, is zero. As the carrier drifts slightly away from resonant, the TEM\(_{01} \) mode acquires an equal phase shift in both photodiode halves while the non-resonant TEM\(_{01} \) mode remains unchanged as shown in Fig. 1(d). This causes the power in each photodiode half to increase and decrease respectively. The electronically subtracted photocurrent is no longer zero and gives an error signal proportional to the phase shift of the carrier.

An earlier system developed by Wieman and Gilbert\(^8\) also uses spatial mode interference. An error signal is obtained with an aperture detector sampling a small section of the beam. A second detector with a variable attenuator samples the entire beam and corrects the offset. This scheme suffers from low efficiency due to the aperture and is limited by the accuracy of the offset compensation.

We present results for two experimental arrangements using the tilt locking technique. The first, shown in Fig. 2(a), obtains the error signal as described above by measuring the light directly reflected from the cavity with a slightly tilted input beam. Figure 2(b) shows double pass tilt locking, where the light passes through the cavity once and is then retro-reflected, with a slight tilt, back through the cavity. The beam reflected on the second pass is used to obtain the error signal.

Single pass tilt locking suffers from the constraint that any input beam displacement causes an offset in the zero crossing point of the error signal. The double pass configuration minimizes this problem by using the first pass through the cavity as a spatial mode filter. In addition the beam path from the retro-reflector to the split detector can be made extremely short and mechanically stable. The error signal frequency response of double pass tilt locking is altered by the addition of an extra pole at the cavity corner frequency, due to the initial transmission through the cavity, giving a frequency response similar to transmission locking techniques.

The split photodetector built for this experiment uses a commercial quadrant photodiode (EG&G C30843E) with the two quarters of each half added together. This forms a vertically split two element detector requiring a horizontal tilt to extract the error signal. The photodetector has sum and difference outputs allowing both the power and error signal to be monitored.

Figure 3 shows experimental results as the cavity length is scanned using a PZT attached to one of the cavity mirrors. Figure 3(a) shows the reflected power and error signal obtained using the single pass tilt locking scheme as the cavity is scanned through a complete free spectral range (FSR). A large error signal (5Vp-p) is obtained even with a small misalignment (TEM\(_{01} \)/TEM\(_{00} \) \sim 1\%). In Fig. 3(a) an error signal also appears (at approximately 8ms) as the small TEM\(_{01} \) mode passes through resonance with the fundamental TEM\(_{00} \) mode. In Fig. 3(b) the error signal appears (at approximately 8ms) as the small TEM\(_{01} \) mode passes through resonance with the fundamental TEM\(_{00} \) mode.

Note that due to the mode cleaner action of the first cavity pass, there is no error signal at the TEM\(_{01} \) resonance. In addition, the error signal drops to zero away from resonance somewhat faster than the single pass case due to the filtering effect of the first cavity pass.
Fig. 3. Power and error signal for (a) single pass tilt, (b) double pass tilt and (c) PDH locking for a cavity with a finesse of 200. Note power in (a) is reflected intensity while (b) and (c) are the transmitted intensities.

For comparison Fig. 3(c) shows the transmitted power and error signal using the PDH technique. The error signal is taken at the output of the demodulation mixer and demonstrates that the size of this signal (0.5Vp-p) is at least an order of magnitude smaller than either of the tilt locking schemes. It should be noted that the size of the tilt locking error signals in Fig. 3 was deliberately reduced to allow use of the same frequency servo designed for PDH locking. Tilt locking error signals of 25Vp-p were readily achieved by increasing the beam tilt.

In order to obtain the error signals of Fig. 3 careful alignment of the laser beam with respect to both the cavity and the tilt detector is necessary. Initially we align the laser beam to the cavity with no offset or tilt. We then position the tilt detector at the centre of the reflected beam by zeroing the subtractor output. By introducing a small tilt of the input beam an error signal is generated. This signal can be adjusted, using fine alignment of the input beam to give a symmetric error signal and thus ensure zero crossing at cavity line centre. Any offset present in the final alignment is apparent by the asymmetry of the resulting error signal and can be readily removed.

Single pass tilt locking and PDH locking have identical frequency responses as both schemes sample the optical field reflected from the cavity. Figure 4(a) shows the frequency spectrum of the error signals of both PDH and single pass tilt locking. The noise features present are typical of our 50 mW Nd:YAG laser (Lightwave model 120) and show the mechanical resonances of the laser crystal. The difference between the two spectra shown here is due to the R-C time constant of the quadrant detector and the limited bandwidth of the audio op-amps used (AD708). A well known feature of the PDH technique is its immunity to laser intensity noise. The intensity noise spectrum of our laser, shown in Fig. 4(b), has a large single feature at 430kHz; the laser relaxation oscillation. This feature is absent from the PDH error signal as expected, and from the tilt locking error signal.

Tilt locking relies on balanced power on the two photodiode halves to obtain this intensity noise immunity. If the power is perfectly balanced then the intensity noise is subtracted down to the quantum noise limit. In practice however, the balancing of the power on the photodiodes, and thus the intensity noise immunity, will be determined by the DC gain of the locking servo. For the servo used in this demonstration we expect approximately 100dB isolation from laser intensity noise.

This method can also be used with other higher order spatial modes with different types of multi-element photodiodes. For example, Gauss-Laguerre interference can be measured efficiently using a bullseye photodiode.

Tilt locking is an inexpensive system requiring only a quadrant photodiode and several low frequency op-amps. For this experiment the total cost of these items was less than $100. In addition, tilt locking can be used with servos designed for PDH locking schemes with no modifications other than a reduction in gain. This combination of low cost, simplicity, and high sensitivity should facilitate the immediate use of tilt locking in a broad range of applications.

This work was supported by the Australian Research Council as part of the Australian Consortium for Interferometric Gravitational Astronomy.