Focusing Light by Diffraction:
Observations and Analysis of Talbot and
Fresnel Zone Plate Images

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Abstract

The diffraction and interference of light are closely related phenomena resulting from the wave nature of light that are familiar to anyone who has studied optics. Less well known is that these phenomena can be used to create images or to focus light, or even x-rays. Creation of “self images” of an object through diffraction is called the Talbot effect, while focusing light with a zone plate - a bull’s eye like pattern of clear and opaque rings of varying width - is named after Fresnel, a French scientist. In this project, the Talbot effect and Fresnel zone plates were studied experimentally with the same setup, which consisted of a fiber-optic light source, a collimating lens, a 1.2 meter long precision optical bench, and a CCD camera. It was discovered that the type of lens, as well as its diameter and focal length, is an important factor for getting good collimation.

Many observations were made of the positions of the zone plate foci and the Talbot images. The measured positions were found to be in excellent agreement with mathematical predictions derived from the known properties of the two diffractive elements.

Future experiments will explore the possibility of creating additional diffractive elements with more specialized properties, like the creation of fractal images.

Research Topic Selection

During summer 2004 I participated in a full-time research program at a university optics laboratory. In this program one is free to choose one’s own research topic. The topic I eventually settled on, a study on focusing light by diffraction, combines my passion for mathematics with a curiosity about an essential physical phenomenon in optics.
1 Introduction

Diffraction and interference are effects that result from the wave nature of light that are familiar to anyone who has studied optics. Diffraction is basically the bending of light around an obstruction, while interference is the interaction between light waves from different sources to produce cancellation or enhancement of intensity. Both effects work together to produce patterns of light called “diffraction patterns.”[1] Less well known is that diffraction and interference can be used to create images or to focus light. Creation of “self images” of an object by diffraction is called the Talbot effect after Henry Fox Talbot (an inventor of photography) who first noticed noticed the effect in 1836. Focusing light with a zone plate - a bull’s eye like pattern of clear and opaque rings of varying width - is named after Fresnel, who developed the theory of zone plates in the early 1800’s. Fresnel zone plates have the fascinating property that they increase light (at the focal point) by removing light (from the blocked zones). The Fresnel zone plate phenomenon and Talbot effect are similar in that the “lens” created by diffraction has multiple focal lengths, unlike an ordinary lens, which has just one.[2]

The Talbot effect and Fresnel zone plates were explored with the same setup. It consisted mainly of the following components: a Helium-Neon laser; a single mode fiber-optic cable; a Fresnel zone plate or Ronchi grating; a collimating lens; and a CCD camera. All of the above were mounted on carriages which could be moved along a Gaertner 1.2 m long precision optical bench.

The main part of my project consisted of a series of experiments with the Talbot effect and Fresnel zone plates to precisely measure and then analyze the distances at which interesting images appeared. With both effects I investigated the results when the incident laser light was either parallel or diverging. The analysis consisted of creating specific mathematical predictions based on the properties of the diffractive elements which were then compared to the measured data.

2 Experimental Setup

Experiments with both types of diffractive elements were done with the same apparatus, which consisted of the following main components: a fiber-optic light source; a precision
optical bench with movable carriages; the diffractive element being studied; and a viewing screen or computerized camera. In addition, in some experiments various lenses where used to collimate (make parallel) the diverging light from the fiber.

The first carriage was always located at one end of the bench and had the fiber-optic cable attached to it. The next carriage either held the lens, which was roughly 4 cm away from the tip of the cable or was not used if the diverging light was needed. The zone plate or Ronchi grating was held by the third carriage and the last held either a viewing screen or an Electrim 1000N CCD camera.

### 2.1 Fiber optic light source

The light source consisted of a helium-neon (HeNe) laser with a wavelength of 632.8 nm directed by mirrors through a single mode fiber optic cable. The fiber was vital to the experiment and determined two crucial details of the experiment. The fiber created a perfect diverging point source of light. Without the cable a lens would have been necessary to diverge the light and could potentially cause error because of a flawed lens or misalignment of the laser beam. The cable acted to create a “clean” light source with a mathematically perfect spherical wave front. The images produced in this setup could have been blurred or difficult to see if the light source was not clean.

### 2.2 Mechanical components

Because both the Talbot effect and the Fresnel zone plates have a large range in which the desired images can occur, one needs to be able to vary the the distance between the various parts of the setup in some precise way. A Gaertner 1.2 m long precision optical bench with four carriages provided the perfect venue to conduct my experiment. The bench was level, meaning that the beam was parallel to the bench and perpendicular to the diffraction gratings, this is an important detail which dramatically affected the entire setup. The bench also allowed precise steady movement of the optical elements. The movements were also able to be measured accurately because the bench itself had millimeter markings. However, the bench did restrict the experiment in one respect because the optical elements had limited motion due to the relatively short length of the bench.
2.3 Collimating lenses

The foci of Fresnel zone plates and half the images of the Talbot effect were produced by collimated light. This means that the light is parallel and in theory the diameter of the beam 10 meters away from the light source will be the same as it was immediately after the source.

Figure 1: Illustration of forming collimated light.

The collimated light in my experiment was created by placing a converging lens exactly one focal length away from the tip of the fiber-optic cable, as shown in Figure 1. I used a plano-convex lens with a 4 cm focal length. Plano-convex describes the shape of the lens meaning that it is flat on one side and spherical on the other. Since the lens was flat on one side the beam was only slightly affected if it was not directed through the exact center of the lens. If the lens was bi-convex the difference in thickness varies quicker leaving almost no room for error when directing the beam of light.

I soon noticed that the lens that was used resulted in a beam with a “hard edge,” in which the intensity cut off suddenly at the edge of the light spot, rather than gradually, as expected. I first hypothesized that the lens was not big enough and was cutting off some of the beam. I tested this by putting a pencil on the edge of the lens to see if this would affect
the edge of the beam. The pencil tip did not affect the beam until it was roughly 1.5 cm inside the edge of the lens. The three next best reasons I could think of to compensate for the hard edge was an abnormality in the lens, internal reflection, or simply that the lens was not strong enough to focus the light. However the hard edge exceeded the boundary of the zone plate and did not appear to have any effect on any images.

2.4 Diffractive elements

Ronchi gratings

Ronchi gratings take their name from Vasco Ronchi, who founded the Italian National Institute of Optics in 1926.[3] Ronchi’s most well known contribution to the world of optics is the Ronchi test.[4] This is a procedure made to test the precision of an optical element like a mirror or lens. The light is propagated though the Ronchi grating and produces fringes on a viewing screen. By analyzing these fringes one can determine the magnitude of aberration of the optical element being tested. A Ronchi grating consists of alternating clear and opaque bands all of equal widths. The Ronchi grating used in the current experiments was 50 × 50 mm and had a period of 10 lines per mm.

Fresnel zone plates

The Fresnel zone plate used in the experiment had 38 zones and a diameter of 17mm. It was printed on a square glass slide and the central zone was opaque. A zone plate with a clear first zone produces a higher intensity focus, but when using light the foci can not be seen by the naked eye. When focusing x-rays and atomic beams zone plates with clear first zones are more common because they result in a higher intensity principle focus that can be measured clearly but not seen.

2.5 Computerized camera

The last carriage held an Electrim 1000N CCD camera. The Electrim imaging computer program allowed me to enlarge the images for better study and to store pictures of the images on the computer.
3 Talbot Effect Studies

3.1 Theory

The creation of “self images” of an object by diffraction is called the Talbot effect, after Henry Fox Talbot, an inventor of photography, who first noticed the effect in 1836.[2] Talbot observed that when a periodic grating was illuminated by a divergent white light source, colored bands appeared at certain distances from the grating. This occurrence that Talbot discovered was not explained until fifty years later when Lord Rayleigh described it as self-imaging in the Fresnel region. Talbot Images occur because of light diffracting on the periodic grating by constructively and destructively interfering at specific distances. These focused images, or Talbot images, occur multiple times at certain distances from the periodic grating.

The equation for finding where these images will occur is strikingly similar to the thin lens equation, except that there are multiple possible values of the focal length \( f \).

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]

The focal lengths are given by

\[
f = \frac{2na^2}{\lambda}
\]

where \( n \) is a half-integer or integer (\( n = 0.5, 1.0, 1.5, 2.0, \ldots \)), \( a \) is the period of the grating, and \( \lambda \) is the wavelength of the incident light.[5]

When the light source is collimated the images are equal distances apart. The formula to find the distance that the images are separated by is as follows:

\[
T_L = \frac{a^2}{\lambda}
\]

\( T_L \) is called the Talbot length and is equal to the focal length of equation 2 when \( n = 1/2,6 \)

Similar to finding the focus of a normal lens, the image on the screen before and after the focus is blurred in an interesting fashion. The images between the Talbot images are fractal patterns. This is easy to observe by the naked eye or by a profile intensity graph of the image.
3.2 Observations and Results

The results of the Talbot images when the light was diverging are summarized in Table 1, which has been placed in an Appendix. To calculate the predicted results I subtracted the inverse of the image distance from the inverse of the focus distance, changing the focus lengths by changing the \( n \) values.

From the table one can see that the expected image distances were very accurate and almost identical to the predicted results. As the object distance decreases the image length increases. The number of experimental data points decreases with the increasing length partly because length restriction of the optical bench. The images were not apparent closer than 14 cm from the grating. A reason for this occurrence could be that the beam was not perfectly perpendicular to the grating or the viewing plane.

As the focused Talbot image distance increased the magnification of the image significantly decreased. This is due to the diverging light. The intensity graphs shown in Figure 2 (on the following page) act to quantify the accuracy of the placement of the receiving plane. If the plane were in the exact location then the graph would only have horizontal and vertical lines, where the crests and troughs would be flat topped and the jumps between them would be vertical lines. The slant in the lines is partly due to the slant of the image. The line of intensity represented is perpendicular to the picture edge instead of the bands of the image.

Adding a carriage holding a lens to collimate the light created fascinating results that were quite different from those described above. I measured where the Talbot images occurred when the grating was at 5 different places. By doing this I expected the data to prove that the Talbot effect occurs cyclically with equal spacing between the images. For each set of data I averaged the distance between the images. The position of the grating is the x-axis and the average spacing is the y-axis in Figure 3. The y-coordinate of the horizontal line is the Talbot length, 1.86, which is where the points are predicted to lie.

With the aid of a spread sheet program I calculated the goodness of the points by squaring the difference between the measure points and the predicted points on the line and then adding these numbers together. This resulted in a goodness of measurement of 3.371.

Figure 4 is more specific in quantifying the accuracy of each individual set of data. The x-axis is the image number and the y-axis is the image position. The slope of each of these lines should be 1.86. The respective slopes of the data for when the grating is positioned at
Figure 2: Talbot effect intensity plots where y-axis increases with the degree of darkness.
Figure 3. Variance between the average measured and predicted data points.

Figure 4. The graph above quantifies the consistency and accuracy of the measured image spacing.
10, 15 and 20 are respectively 1.93, 1.93 and 1.75.

The Talbot effect is related to many topics in mathematics and theoretical physics. “Quantum revivals,” for example are related to the cyclic images of the Talbot effect. The wave packet corresponding to a particular particle starts in a certain form and then returns to that form periodically. The Talbot effect is also related to fractal imaging in mathematics. At intermediate points between the Talbot images fractal bands occur. The effect can be applied to classical and atomic optics, in that it can be used to focus atomic beams.

4 Fresnel Zone Plate Studies

4.1 Theory

Fresnel zone plates are optical elements in the field of diffractive optics. They behave in many ways as normal optical lenses would because they are capable of focusing light. They consist of concentric circles which form the boundaries of concentric regions known as zones. For a typical zone plate half of these zones are transparent to light while the alternate zones are opaque, and block light. This makes the zone plate appear to have a “bulls-eye” pattern where the first or central zone of the zone plate can be either opaque or clear. Fresnel zone plates have the fascinating property that they increase light at the focal point by removing light at the blocked zones.[7]

Glass lenses, no matter how perfect, have certain limitations that Fresnel zone plates can overcome. For example, Fresnel zone plates can be used to focus x-rays or beams of atoms. When an x-ray is propagated onto a glass lens it is not refracted because the index of refraction of glass is 1.0, and moving atoms are simply not able to penetrate glass materials.

If the zones had arbitrary sizes and/or positions a zone plate would have no affect on an incident beam of light other than to reduce the intensity of the beam. However, zone plates are constructed in a very specific way so that they are capable of focusing an incident collimated beam of light by diffraction. One way of stating this is that each zone must enclose an equal area.[8] Because of this property the radii follow the relation below, where \( r_1 \) and \( r_2 \) are two consecutive zone radii:

\[
\pi r_1^2 = A_{\text{zone}}
\]
\[ \pi(r_2)^2 - \pi r_1^2 = A_{\text{zone2}} \]

\[ A_{\text{zone1}} = A_{\text{zone2}} \]

Therefore,

\[ \pi r_2^2 - \pi r_1^2 = \pi r_1^2 \]

or

\[ \sqrt{r_2^2} = \sqrt{2r_1^2} \]

which simplifies to

\[ r_2 = r_1\sqrt{2} \]

and is in general form:

\[ r_n = r_1\sqrt{n} \]

So one can see that the \textit{nth} radius is equivalent to the product of the first radius times the square root of \textit{n}.

Each zone is separated by \( n\lambda \) (\( n \) wavelengths of light) from the focus. This causes the light to diffract in such a way that a focus is produced on the viewing screen if set at the right distance.

Another interesting property of zone plates is that they have multiple focal points or foci. The principal focus has the greatest intensity and is farthest away, as the foci get closer the intensity decreases because light is not diffracting around all of the zones. Conversely because light is being diffracted by all the zones at the farthest distance, that focus the most intense.

There is a specific formula to calculate the focal \textit{distances} of a zone plate (the distances the receiving screen must be placed at in order to view a focus). The formula follows from the thin lens formula:
\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]

and this standard expression for the focal lengths of the zone plate

\[
f = \frac{2\pi \lambda}{nr^2}
\]

to fi.

This formula will yield the distance of the principal focus. The secondary foci after that are \(f/3, f/5, \ldots, f/a\) away from the zone plate, when \(a\) is an odd integer.

### 4.2 Observations and Results

I measured the distances of where the zone plate foci formed in a similar fashion described previously for finding Talbot images.

The average measured principle focus of the Fresnel zone plate is 53.67 cm. This is completely different from the predicted measurement of the principle focus and from the table below one can see that the relationship of the principle focus to the other foci is clearly not of the form \(f/n\), where \(n\) is an odd consecutive integer. After analyzing the data carefully, I then decided to test if 54 cm really was the principle focus because I suspected that there could have been additional foci beyond the 1.2 m limit of the optical bench. So I first let 54 be the second focus meaning that \(54 \times 3 = f\) and \(f/5 = 38\), but this did not prove true. I then let 54 be the third focus so that \(54 \times 5 = f\) and \(f/7 = 38\) which was accurate. This meant that 270 cm was the principal focus, 90 cm was the second focus and the focus that I thought was the first was actually the third.

Figure 5 quantifies this result. The \(x\)-axis is the number which the principle focus was divided by and the \(y\)-axis is the inverse of that number. The red line is where the ideal data points would lie.

I attempted to then find a zone plate image when the light was diverging. No clear foci formed on the viewing screen. Following with the comparison of zone plates to lenses I think that the zone plate is not a strong enough lens to focus the diverging light. It was able to focus light that was only slightly diverging, which was produced by a lens. However, the zone plate was not powerful enough to focus the light diverging from the fiber optic cable.
<table>
<thead>
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<th>Zone Plate Camera Position and Image distances</th>
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<tr>
<td>10</td>
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<td>17.3</td>
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<td>15.2</td>
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Table 2. Measured zone plate focal distances (cm) for varying positions of the zone plate.

Figure 5: Analysis of zone plate image distances.
The other probable reason why there were no apparent foci could be that the foci formed beyond the distance of 1.2 m from the cable.

5 Further Work

There are a number of questions and topics that have come up in the course of my work so far, and which I hope to address in the near future.

Perhaps the most important issue that needs to be addressed is that of understanding and improving the collimation of the laser light incident on the grating.

I would also like to continue to analyze the “lens properties” of the Fresnel zone plate. For example, I want to find a way to control the divergence of the beam, and find the maximum divergence of the beam when a focus will still form. From this data I would be able to calculate the “F/ number” of the zone plate and define its properties as a lens. (The F/ number – a term which is frequently used in photography to describe the light gathering power of a lens – is defined as the ratio of the focal length to the lens diameter.)

A more ambitious future interest is to construct my own zone plates. For example, these zone plates could have zones in a fractal pattern. It would be very interesting to see how fractal zones would affect the focusing properties of the light. I would also like to investigate if some zones were more significant in the focusing properties than other zones. I would do this by making clear zones opaque one at a time.

With the Talbot effect I would like to duplicate and extend Talbot’s original experiment with white light. It would be fascinating to see if different types of white light produce variations in the bands of color that would form. I would also like to examine the Talbot effect, if possible, with a grating that has zones with a similar relation to the zone plates.

6 Conclusions

Many observations were made of the positions of the zone plate foci and the Talbot images. The apparatus produced very precise data which, after analysis, were found to be in excellent agreement with mathematical predictions derived from the known properties of the two diffractive elements. In one case it initially appeared that there was a serious discrepancy between data and predictions, but this difficulty was traced to a mis-numbering of the focal
points and resolved.

In the course of the experiments it was discovered that the type of collimating lens, as well as its diameter and focal length, is an important factor for getting good collimation. This topic and several others described in the previous section are the subject of my continuing and future experiments.

References


Appendix A: Summary of Talbot Effect Data

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<th>Image Distances (cm) for Specified Object Distances</th>
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Table 1. Predicted and experimental image distances (cm) for varying object distances