Optical phase singularities in detection of laser beam collimation

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Optical phase singularities, also called optical vortices, are known to have applications in various branches of optics. Here the role played by optical vortices in collimation testing is explained. Interference and a diffractive experimental setup in which the presence of an optical vortex permits collimation testing are presented. It is shown that the moiré fringes that aid in collimation detection are due solely to the presence of vortices and not to the accompanying phase factors that are involved in producing a grating structure. © 2003 Optical Society of America

OCIS codes: 120.1680, 120.2650, 120.3180, 120.3940, 120.4120.

1. Introduction

Wave fronts that have helical structures with undefined phase at an isolated zero amplitude point are called optical phase singularities or optical vortices. Optical vortices have been receiving much attention recently because of their unique properties. Vortices carry orbital angular momentum and are studied for their roles as vortex solitons and in various other applications. In most of optical experiments and in the study of optical systems it is necessary to use collimated beams. The degree of collimation has been determined mainly by shearing interferometry, which provides the phase gradient maps, and by Talbot interferometry, which is a combination of a self-imaging phenomenon and the moiré effect.

In this paper the role of optical phase singularities in collimation testing, which so far has been unclear, is explained. Ways in which interference and diffraction experiments can be set up for collimation testing by use of optical vortices are explained. A brief explanation of the principles involved in collimation testing and a note on optical vortices followed by an explanation of the role played by optical vortices in collimation testing are presented.

2. Collimation Testing

In collimation testing the path function $W$ is represented by

$$W(x, y) = D(x^2 + y^2),$$

where $D$ is the defocus for an otherwise aberration-free wave front. For a lens of focal length $f$, if the point source at its focus is displaced by a distance $\Delta f$, the defocus is given by

$$D = \Delta f/2f^2. $$

To detect the defocus, one forms interference fringes with a suitable reference wave. For example, when a plane wave is used as the reference wave, a non-collimated beam produces circular fringes, where

$$\Delta W = D(x^2 + y^2) = n\lambda,$$

$n$ represents the interference fringe order, and $\lambda$ is the wavelength of light. But this is an inefficient way to detect defocus for two reasons, viz., (1) one needs a plane reference wave to detect a plane wave and (2) one cannot effectively use this method to detect beams that have low divergence. One of the methods for detecting beams with low divergence uses the gradient of the path function, where

$$\Delta W = \frac{\partial W}{\partial x}\Delta x = 2Dx\Delta x = n\lambda.$$

Here the test beam itself provides the reference beam. $\Delta x$ is the infinitesimal increment in $x$ that represents a lateral shift of the test wave front. The sensitivity of this method has been improved remark-
ably by various methods in which an additional tilt is given to one or to many of the interfering beams or by obtaining dual fields that produce fringes that evolve in opposite ways when collimation is disturbed. With an additional tilt to one of the beams, the interference condition reads as

$$\Delta W = \frac{\partial W}{\partial x} \Delta x \pm y \beta = n \lambda.$$  \hspace{1cm} (5)

In dual fields the opposite tilt components (±yβ) are restricted inside two juxtaposed fields in fringe formation, whereas in interlaced fields the tilted fields overlap. There is also a possibility of using the direction of phase gradient and tilt either orthogonally or parallel to each other.

In diffraction methods for collimation testing\textsuperscript{10–12} the change of curvature of a wave front on propagation is detected. One can detect this change in curvature by producing moire fringes in a Talbot interferometer that evolve differently for collimated and for noncollimated beams. A linear grating of size LXL with a transmittance function given by

$$t(x, y) = \left[ \text{rect} \left( \frac{x}{a} \right) \delta(x - 2pa) \right] \text{rect} \left( \frac{x}{L} \right) \text{rect} \left( \frac{y}{L} \right),$$  \hspace{1cm} (6)

where a is the width of individual slits, self-images under collimated illumination at planes located at Z\textsubscript{N}, where

$$Z_N = N/\lambda \mu^2.$$  \hspace{1cm} (7)

N corresponds to the N\textsuperscript{th} self-image plane, λ is the wavelength of the illuminating beam, and μ is the frequency of the grating. The self-image planes are equidistant, and the self-imaged gratings have same period. When the grating is illuminated with a non-collimated beam, the frequency of the self-image changes, and frequency difference Δμ is given by

$$|\Delta \mu| = N/\lambda \mu R,$$  \hspace{1cm} (8)

where R is the radius of curvature of the beam and the self-images are not equidistant. By placing suitable gratings at the self-image planes, one can form moire fringes. The evolution of fringes when the collimation of the beam is disturbed is used for collimation testing. In what follows, a brief outline of optical vortices is presented for derivation of the path functions required for collimation testing when vortices are involved.

3. Optical Vortices

In free space, a linearly polarized monochromatic optical field

$$U = (x \pm iy)^m E_0 \exp(-ikz),$$  \hspace{1cm} (9)

which satisfies the scalar wave equation, describes a single optical dislocation\textsuperscript{13–15} with the magnitude of the topological charge given by m. E\textsubscript{0} in Eq. (9) is a constant. The phase distribution is given by

$$\Theta(x, y) = \text{arg}[(x \pm iy)^m] - kz.$$  \hspace{1cm} (10)

In the complex field given by Eq. (9) the core of the vortex is located at the origin, where the phase is undefined. It has an isolated zero-amplitude point with spiral phase singularity where all phase values from 0 to 2mπ can be found in an arbitrary small region about the point. The circulation of the gradient of phase function ∇\Theta about a sufficiently small positively oriented closed path C enclosing the vortex center is 2mπ, where topological charge m is given by

$$m = \frac{1}{2\pi} \oint_c \nabla \Theta \cdot dl.$$  \hspace{1cm} (11)

An isolated zero location can simply be a zero, however, and does not necessarily lead to a spiral phase singularity, in which case \(\oint_C \nabla \Theta \cdot dl = 0\). Independently of the direction of the way through the singular point with |m| = 1 (vortex core) there is a phase jump of \(\pi\), and at the singular point the phase becomes indeterminate. The equiphaseline for a vortex appear to start from or end at the core of the vortex, forming a starlike structure. This is different from the wave fronts encountered in optical testing, in which the phase difference between any two points (say a and b) can be computed independently of path on the wave front; i.e., line integral \(\int_a^b \nabla \Theta \cdot dl\) is independent of path. The presence of optical vortices makes the line integral strongly path dependent.

Figure 1(a) shows a negatively charged vortex phase distribution with charge \(m = -1\), and Fig. 1(b) shows a positively charged vortex phase distribution with charge \(m = +2\). As the phase of a complex field must be single valued, the topological charge of an optical vortex can take only integer values. In these figures, all phase values from 0 to 2π are represented by gray levels—from dark to bright.

4. Collimation Testing with Vortices

A. Phase Plate

When a spiral phase plate with phase distribution given by Eq. (10) is introduced into a beam, the path function is given by

$$\Delta W = \frac{1}{k} \Theta(x, y).$$  \hspace{1cm} (12)

When this beam is made to produce interference fringes with a plane wave, the fringe formation is governed by

$$\frac{1}{k} \Theta(x, y) + y \beta = n \lambda.$$  \hspace{1cm} (13)

This is the well known fork grating type fringe with tilt \(\beta \neq 0\) in the interfering plane wave. Tilt com-
ponent xβ instead of yβ can also be used in Eq. (13), which will result only in a change in the orientation of the fringes. Formation of such fringes plays an important role in the detection and generation\textsuperscript{16–18} of an optical vortex in a light beam. When β = 0, there are m dark fringes radiating from the core center that thus form a starlike structure. Figure 2(a) shows the interference pattern when β ≠ 0, for which two (for \( m = 2 \)) dark fringes are terminated in the middle of the interference pattern, and Fig. 2(b) shows the case for β = 0. For collimation testing with β = 0 the fringe formation is governed by

\[
\Delta W = \frac{1}{k} \Theta(x, y) + D(x^2 + y^2) = n\lambda. \quad (14)
\]

The presence of a quadratic phase factor (the result of defocusing) in the equation results in spiral fringes with \( m \) arms evolving from the core center. Figure 3 shows the case for \( D \neq 0 \) and \( m = 2 \). As described above, the use of a plane wave as a reference wave is not appropriate for collimation testing. Hence to use the test beam as the reference beam we consider the Twyman–Green setup shown in Fig. 4.

The laser beam under test is divided into two at the beam splitter, and the two arms of the interferometer are made to have unequal arm lengths but a path difference that is less than the coherence length of the light being tested. A spiral phase plate is introduced into one of the arms of the interferometer. It is difficult to fabricate a spiral phase plate, which is a continuous phase element; hence a liquid crystal can be used for this purpose. The two beams are combined by the same beam splitter, and fringes can be observed at the output port of the interferometer. At collimation, the unequal arm lengths of the interferometer produce 2\( m \) radial fringes. When the collimation is disturbed,
spiraling of fringes occurs, and the fringe formation condition just after the beam combiner is given by

\[ \frac{2}{k} \Theta(x, y) + (l_1 - l_2) \left( 1 + \frac{x^2 + y^2}{2l_1l_2} \right) = n\lambda, \quad (15) \]

where \( l_1/2 \) and \( l_2/2 \) are the arm lengths of the interferometer. The second term describes the path difference between the two beams, which has different sagittae as a result of unequal lengths of the arms in the interferometer. The second term will be zero for a collimated beam and also for \( l_1 = l_2 \). The factor of 2 that appears in the first term of Eq. (15) is due to the fact that the interferometer is a double-pass interferometer.

Note that at collimation the fringes form a starlike structure and that at noncollimation they form a spiral structure. Figures 5(a), 5(b), and 5(c) show the fringe patterns for \( D < 0 \), \( D = 0 \), and \( D > 0 \), respectively.

B. Grating

Fabrication of a spiral phase plate is difficult; hence diffractive methods can be used for collimation testing with optical vortices. Consider a grating with a transmission function given by

\[ t_{1s}(x, y) = \frac{1}{2} + \frac{1}{2} \cos \left[ \Theta(x, y) + \frac{k}{R} (x^2 + y^2) \right]. \quad (16) \]

Here \( k \) is a constant that has the dimensions of a propagation constant. When this grating is illuminated, self-images are formed at Talbot planes whose spacing is equal for collimated light and unequal for noncollimated light. Also, the period of the grating at the self-image plane is the same as that of the original grating that is illuminated for collimated light, whereas it is different for noncollimated light. Figure 6 shows the Talbot interferometric setup in which the self-image planes are separated by distances \( Z_N \) from each other, as governed by Eq. (7). At the self-image plane we can write the field distribution as

\[ t_{1s}(x, y) \equiv \frac{1}{2} + \frac{1}{2} \cos \left[ \Theta(x, y) + \frac{k}{R} (x^2 + y^2) \right]. \quad (17) \]

At this plane, if another grating with transmittance given by

\[ t_2(x, y) = \frac{1}{2} + \frac{1}{2} \cos \left[ \Theta(x, y) - \frac{k}{R} (x^2 + y^2) \right] \quad (18) \]

is placed, the field just behind the second grating is given by

\[ a(x, y) = t_{1s}(x, y)t_2(x, y). \quad (19) \]

The intensity distribution is given by

\[ I(x, y) = I_0(x, y) + \alpha I_1(x, y) \]

\[ = I_0(x, y) + \alpha \left\{ \cos \left[ \Theta(x, y) + \frac{k}{R} (x^2 + y^2) \right] \cos \left[ \Theta(x, y) - \frac{k}{R_S} (x^2 + y^2) \right] \right\}, \quad (20) \]

where \( I_0 \) includes terms that may not be important in moire fringe formation and \( \alpha \) is a constant that shows that bracketed term \( I_1 \) occurs more than once in the expression for intensity distribution.

The bracketed term can be written as

\[ I_1(x, y) = \frac{1}{2} \left\{ \cos \left[ 2\Theta(x, y) + k(x^2 + y^2) \left( \frac{1}{R} - \frac{1}{R_S} \right) \right] \right. \]

\[ + \left. \cos \left[ k(x^2 + y^2) \left( \frac{1}{R} - \frac{1}{R_S} \right) \right] \right\}. \quad (21) \]

At collimation \( R = R_S \) the moire fringes are radial, and at noncollimation \( R \neq R_S \) the moire fringes are spiral.

The transmittance function given by Eq. (16) for the grating consists of two parts inside the cosine modulation. One part corresponds to vortex phase; the other, to quadratic phase. In the experiment described above, the radial moire fringes at collima-

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Fig. 3. Fringes in Fig. 2(b) spiral when collimation is disturbed \((m = 2)\).

Fig. 4. Interferometric setup for collimation testing: M1, M2, mirrors; BS, beam splitter; P, spiral phase plate.
tion are due to interference between two vortex-infested wave fronts with opposite topological charges. These radial fringes are made to spiral by the presence of the term

\[
(l_1 - l_2) \left(1 + \frac{x^2 + y^2}{2l_1 l_2}\right)
\]

in interference and by the presence of the term

\[
k(x^2 + y^2) \left(1 - \frac{1}{R_1 R_2}\right)
\]

in the diffraction experiment. They become nonzero for noncollimated beams. The other terms in the intensity distribution of Eq. (19) in the Talbot experiment do not contribute to the spiraling because some of the cross terms indicate cancellation of vortices and individual terms merely reproduce the grating structure. It is worth mentioning here that interference of two vortex-infested beams of like charge results in an intensity distribution with no fringes and that interference of beams that have opposite charges results in fringes radiating from the vortex core when the cores coincide. Interference is an addition operation in which the phase difference results in fringes,
in contrast to a multiplication operation in which opposite charges annihilate each other.

The transmittance function given by Eq. (16) is the intensity pattern that is due to the interference of two waves, one with a quadratic phase and the other with a vortex phase, but it is also possible to use a linear phase given by $\exp[ib(x^2 + y^2)^{1/2}]$ instead of a quadratic phase, in which case the grating will have uniform fringe spacing. This phase variation is linear in $r$, so the wave front has the shape of a cone ($r$ is the distance from the optical axis where the vortex core is located in the transverse plane). Figure 7(a) shows the amplitude grating structure modulated by the quadratic phase term of Eq. (16), and Fig. 7(b) shows the amplitude grating structure with the linear phase mentioned above. It is the optical vortex that plays a crucial role in the detection of collimation in these experiments, and the grating structure can thus be modified to produce good Talbot self-images. Figure 8(a) shows the moire fringes when quadratically modulated amplitude gratings with different periods are superimposed. Figure 8(b) shows the case for a linear grating. The moire fringes in the two cases are similar, even though the gratings that produced these fringes have different transmittance functions. In these cases the topological charge of the optical vortex is invariant under propagation.

The number of arms in the fringe patterns obtained in these arrangements indicates the sum of the magnitudes of the topological charges that are involved (or encountered by the beam) in the fringe formation. This means that interference fringes–moire fringes with odd numbers of arms can also be produced and used for collimation testing by suitable arrangements. Figures 9(a) and 9(b) show examples of such a possibility for $D = 0$ and $D \neq 0$, respectively. All the simulations in this paper were made with VOL4 virtualLab software from LightTrans GmbH.
the number of fringes and the topological charges and the ways in which accompanying phase factors modify the gratings for diffraction experiments have been treated.

References


Fig. 9. The number of arms in the fringes indicates the number of optical vortices that are present in the two elements put together. An odd number of fringes is shown when (a) \( D = 0 \) and (b) \( D \neq 0 \).

5. Conclusions

Optical fields that possess phase singularities have applications in various fields. In this paper the role played by optical vortices in collimation testing has been described. In collimation testing with optical vortices, one can detect the change of curvature of a noncollimated beam by observing the evolution of star fringes in both interference and diffraction experiments. The fringes that aid in the detection are due purely to the presence of vortices and not to the accompanying phase factors that are involved in producing a grating structure. The relation between