Moiré Topography

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A technique observing contour lines of an object by the use of moiré is developed. Shadow of an
equispaced plane grating is projected onto an object by a point source and observed through the
grating. The resulting moiré is a contour line system showing equal depth from the plane of grating
if the light source and the observing point lie on a plane parallel to the grating. A technique to wash
away the unwanted aliasing moiré optimization of contour line spacing and visibility and the results
of the application are described.

The best way to describe a three-dimensional shape
is to draw the contour lines. Formation of the Newton's
ring between an object and an optical flat is a direct
means of observation and recording of contour lines,
but the depth of the surface under test is limited to not
more than several tens of the wavelength of light used.
There has been no simple way of observing contour
lines of an object with greater depth.

Tsuruta\textsuperscript{1} proposed an ingenious way of recording
contour lines of an object with a diffusing surface.
This method, however, visualizes contour lines only
after several processes and requires high quality optics.
The size and depth of the test object may be limited.

Drawing contour lines from a pair of stereo pictures
with the aid of a drawing machine is most commonly
used to obtain contour lines of a large object but, like
the aforementioned method, this is not a direct method
either and requires expensive instruments.

Multislit \textit{Lichtschnittverfahren} with 90-deg incidence
angle visualizes contour lines \textit{in situ} but only on convex
surface.

This paper will describe a method for visualizing con-
tour lines \textit{in situ} on an object of medium (such as a face
of a coin) and large (such as a car) size and depth.

Suppose an equispaced plane grating with line spacing
$s_0$ is placed over an object to be tested (Fig. 1). The
surface is illuminated by a point source $S$ and ob-
served through a small hole at $E$. The $x$ coordinate is
taken to lie along lines of grating and the $z$ coordinate is
taken to be perpendicular to the grating surface. $S$
and $E$ lie on the $Y-Z$ plane. The shadow observed from $E$
around $R$ is the result of central projections applied twice on the grating around $Q$. Because the line spacing $s_0$ is very small compared
with vertical distances $S$ and $E$ from the grating, the
projections are practically parallel projections. There-
fore the shadow observed through grating around
$R$ is a grating with different direction and spacing from
the original grating but with same phase relation to $R$
with that of original grating to $Q$.

Assuming a sinusoidal grating, transmittance of the
grating is

$$T_Q = \frac{1}{2} [1 + \cos 2\pi (\epsilon + y)/s_0],$$

(1)

where $s_0$ is line spacing of the grating and $\epsilon$ is initial
phase of the grating.

The projection of shadow around $R$ is

$$I_s = \frac{1}{2} [1 + \cos 2\pi (\epsilon + y_Q)/s_0 + z/s']I_0,$$

(2)

where $y_Q$ is $y$ coordinate of $Q$, $\xi$ is a new coordinate
with the origin on $R$, which is vertical to the projection
of shadow of the grating lines on $xy$ plane. Using $x$
and $y$, $\xi$ is expressed as follows, assuming the angle of
projected shadow relative to original lines as $\theta$, and the
line spacing of projected shadow as $s'$:

$$\xi = (y - y_R) \cos \theta - (x - x_R) \sin \theta.$$

(3)

Transmittance of the grating around $R$ is expressed
by $T_R$, which is represented by the same form as Eq. (1).
The moiré observed around $R$ is obtained as a product of $I_s \times T_R$; thus

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Fig. 1. Schematic representation of moiré topography. G is a equispaced plane grating. The X axis is taken along with lines of grating and the Z axis is taken vertical to the grating surface. E is the view point and lies on the Z axis. S is a point source which lies on the Y-Z plane. They coordinate of S is \( s_x, s_y \). Heights of \( E \) and \( S \) from the grating surface are \( 1E \) and \( 1S \). Line spacing of the grating is \( s_0 \) and \( \epsilon \) is the initial phase of the grating. \( P \) is a point on a surface under test. Depth of \( P \) from the grating surface is \( h \), which is taken positive to downward. The slightly curved dotted lines represent the shadow of a small area of grating around \( Q \) and the slightly curved thick lines represent a perspective from \( E \) of the shadow on the grating plane. The perspective is approximately a grating with line spacing \( s' \) and line direction \( \theta \).

The coordinates of \( Q \) and \( R \) are \( X_Q, Y_Q, X_R, Y_R \).

\[
I_M = \{1 + \cos \pi [(x + y)/s_0 + (y - y_R) \cos \theta/s'] - (x - x_R) \sin \theta/s' \} \cos \pi [(x + y)/s_0 + (y - y_R)(\cos \theta/s' - 1/s_0)] - (x - x_R) \sin \theta/s' + \cos \pi [(2x + y_R + y)/s_0 + (y - y_R)(\cos \theta/s' + 1/s_0) - (x - x_R) \sin \theta/s'/] / 2 \} / 4. \quad (4)
\]

The first cosine term represents the projected shadow of the grating. This is not necessarily of high frequency, as will be mentioned later. In most cases, however, the frequency of this term is in about same order with that of grating and can be separated from moiré. The second cosine term represents grating itself and the fourth cosine term the sum of grating and its shadow. These are also of high frequency.

The third cosine term represents moiré in the small area around \( R \) on the grating or around \( P \) on the surface. Lightness of moiré at \( P \) is obtained by making \( x = x_R \) and \( y = y_R \), where \( x_R \) and \( y_R \) are \( x \) and \( y \) coordinate of \( R \):

\[
I_P = \{1 + \cos \pi [(y_Q - y_R)/s_0] / 4 \} I_M. \quad (5)
\]

Note that the initial phase of the grating is dropped from the expression. This means that the moiré is stationary against parallel movement of the grating in its plane.

Elementary geometry gives

\[
y_Q - y_R = (l_s - l_b) y_R h / l_b (l_s + h), \quad (6)
\]

where \( h \) is depth of \( P \) from the surface of grating taken positive to downward. By making \( l_s = l_b = l \), Eq. (6) is simplified as

\[
y_Q - y_R = h d / (l + h), \quad (7)
\]

which is a function of \( h \) only for given \( d \) and \( l \).

Thus the equal brightness line represents the equal depth line. The depth of the \( N \)th bright line is obtained by making the argument of the cosine term of Eq. (5) equal to \( 2N \) as follows:

\[
h = N / (d/s_0 - N). \quad (8)
\]

In most cases \( d \) is much larger than \( s_0 \) and the following relation may be used as a good approximation:

\[
h \approx s_0 N / d. \quad (9)
\]

When \( l \) is infinity, which means illuminating with collimated light and observing vertically through a small hole on the focal point of a field lens, Eq. (8) is written as

\[
h = s_0 N \cot \theta, \quad (10)
\]

where \( \theta \) is incidence angle of illuminating light.

Figure 2 shows contour line system of a 25 cent coin, on which a thin coat of white paint is applied. Collimated light and a field lens are used for illumination and observation. The grating used is plane glass Ronchi ruling of one to one ratio of width and space.*

Use of a collimating lens and a field lens is not prac-
Fig. 3. Contour line system of a cotton cloth disk of 36-cm diam revolving with flatter. One turn of a helical Xe flash is used for illumination. $a = 1.0$ mm, $l_\theta = l_g = l = 100$ cm, $d = 20$ cm, $\Delta h = 5.0$ mm. The parts of the disk with fringe of greater visibility are close to the grating.

The visibility of the contour line with greater depth is a function of line width and spacing of grating, penumbra of line, diffusion of light at the object surface, diffraction of light by the grating, and aperture of the observing system. Detailed analysis of this problem is now under way. Only a qualitative explanation and a practical technique to obtain fine contour lines with good visibility for a deep object will be noted.

The essentials for obtaining good visibility are to make distinct bright and dark shadow stripes on the object surface and to observe the stripes with a small aperture so that the bright stripes are blocked by the lines of grating.

Too fine a grating causes diffraction of light which blurs the shadow. Thus this technique is not suitable for observing a very small depth which would be observed successfully by the Newton's ring.

Even if the grating is coarse, the shadow is blurred by penumbra, which is determined by the width of light source. The smaller or thinner the light source along the line of grating the sharper is the shadow.

Light should not be diffused at the object surface; i.e., should be reflected from object surface layer which is as thin as possible. A rough metallic surface usually gives good results. A translucent object needs white surface coating. A small amount of black pigment mixed in the coating improves the visibility because of the improved covering power.

The aperture of the camera lens should be small to increase the depth of focus. A lens of short focal length allows usage of smaller $F$ number than a lens of longer focal length at the sacrifice of image quality.

Larger illumination angle gives finer contour lines using a grating of some line spacing, but too much oblique illumination is not recommended because it gives an unnatural shadow of the object.

Our present system gives visible contour line of 2-mm interval down to 400 mm deep using a grating with 1-mm pitch.

An object which has a surface with large inclination introduces some difficulties. First, the width and spacing of shadow of lines of grating increase on such a part of surface with a small angle to the illuminating light. This means that frequency of the first term of Eq. (4) is decreased and mixed up with the moiré. Second, an aliasing moiré appears, which is actually a moiré formed between higher harmonics of shadow and grating or higher harmonics of grating and shadow.\(^2\) This aliasing moiré does not appear in Eq. (4) because the equation is derived assuming a sinusoidal grating. These unwanted patterns change their shapes and positions with the movement of grating in its plane, whereas the equal depth moiré is stationary. So by moving the grating in its plane during exposure, these unwanted patterns are washed away.

Figures 4 and 5 show contour line system of a mannequin taken with a stationary grating and with a moving grating.

By using two light sources arranged symmetrically about $E$, shadow-free illumination is possible without affecting the contour line system. Figure 6 is a contour line system of a mannequin taken by shadow-free illumination.

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To obtain the moiré system with good visibility of a translucent object, such as the human body, application of liquid powder, preferably darkened down to Munsell

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Fig. 4. Contour line system of a mannequin of living size. $s_0 = 1.0 \, \text{mm}, l_g = l_w = l = 200 \, \text{cm}, d = 100 \, \text{cm}, \Delta h \approx 2.0 \, \text{mm},$ and vertical distance of white lines = 10.0 cm. The grating is stationary during exposure. Note shadow of lines of grating on left cheek, and aliasing moiré on left and right cheeks.

Fig. 5. Contour line system of a mannequin of living size. Data are same as for the Fig. 4 but the grating is moved parallel in its plane during exposure. Note that shadow of lines of grating and aliasing moiré are washed away.

Fig. 6. Contour line system of a mannequin taken by shadow-free illumination. $s_0 = 1.0 \, \text{mm}, l_g = l_w = l = 200 \, \text{cm}, d_1 = d_2 = 50 \, \text{cm}, \Delta h \approx 4.0 \, \text{mm},$ moving grating.

by adding black pigment, is recommended. Figure 7 shows a contour line system of a living human body. Moiré produced between two systems of contour line shows up the contour line system of equal depth difference of the two surfaces.\(^3\)

The two contour line systems are expressed as

$$f_1(x,y) = I\Delta h$$

$$f_2(x,y) = J\Delta h$$

where $I$ and $J$ are integral numbers and $\Delta h$ is depth interval between successive contour lines. The moiré is expressed by a function which satisfies $J - I = K,$ where $K$ is another integral number. The moiré is expressed as follows:

$$f(x,y) = [f_1(x,y) - f_2(x,y)] = K\Delta h,$$

which means that the moiré forms a set of curves of equal depth difference.

Figure 8 shows subtractively engaged two contour line systems of the human back with raised and lowered right arm. Faint but observable moiré of the two contour line systems are observed on such part where orig-
objects. A large field of application of this technique is expected, however, because of its simplicity in theory and practice.

Recently Brooks and Hefflinger published a paper on moiré gauging.4 In their method an equispaced laser interference pattern is projected on a test object, a picture of the pattern is taken, and the negative plate is restored in its original position. Observing the test object illuminated by the same interference pattern through the master negative shows up moiré corresponding to change in the test object.

A main difference between our method and theirs is whether a pattern projected on a test object and a pattern to interfere with the first pattern are dependent, as ours are, or independent, as theirs are, so theirs has more freedom than ours. For instance, if an equispaced linear grating is used as a master negative of their method, the original contour lines are close and the depth change is relatively small. The contour lines cross with large angle on the part where deformation is large and the moiré is hard to observe visually. The cross points with same K's must be picked up to draw contour lines of equal depth difference.

Figure 9 shows a graphically drawn contour line system of equal depth difference, which is made by making an enlarged transparency of the contour lines of before and after deformation, putting the order of the lines I and J to contour lines of each enlarged transparency, putting one on the other, and picking up such points with the same order difference and connecting them in a smooth line.

Application of this technique of direct observation and photographic drawing of contour lines by the use of moiré is limited in range to medium to relatively large

Fig. 7. Contour line system of a living body. $s_0 = 1.0$ mm, $l_x = l_y = l = 200$ cm, $d = 40$ cm, $\Delta h = 5.0$ mm, moving grating.

Fig. 8. Subtractively engaged two contour line systems of back of a living body. Each of the contour line systems is taken with right arm in raised and lowered positions. Note the moiré running near backbone on right shoulder.
moiré will be a contour line system of equal depth from a plane corresponding to the linear grating used.

Because the image of the first pattern must be resolved on the master negative plane, the depth of a test object of their method will be limited. This is also true of our method since sharp shadow of the lines of grating must be projected on the surface of a test object, but the images of lines of grating and their shadow are not necessarily resolved at the image plane, so the requirement imposed on the imaging system is much less for our method. Moreover, coherent light is not necessary, and the moving grating method is easily applied. The observation of deformation of our method is not as neat as theirs. Therefore both methods seem to have their merits and demerits.

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References

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