

# A Magneto-Optical Tachometer Based on the Faraday Effect

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## Summary

The optical phenomena known as the Faraday effect was studied in the context of a particular application. In the Faraday effect, the orientation of the electric field of light - its polarization - is rotated when subjected to a magnetic field. This magneto-optical effect was used to construct a tachometer - a device that measures the rotational speed of a wheel - without electrical components. The tachometer has a variety of applications over traditional electrical sensors.

## Abstract

This research focused on the study of the Faraday effect and its use to construct a magneto-optical tachometer. In the Faraday effect, applying a magnetic field to certain materials changes the polarization of light passing through the material. Some applications of the Faraday effect in addition to the tachometer are optical current sensors and Faraday isolators.

The tachometer measures changes in the polarization of laser light as a piece of magnetically permeable material attached to a rotating shaft modulates the magnetic field in a specific type of Faraday material. To get the greatest degree of polarization change, special rare-earth garnets were used. I have worked with a Terbium Gallium Garnet (TGG) crystal, and have recently begun using a smaller piece of Yttrium Iron Garnet (YIG).

In preliminary experimentation, a 15.1 degree polarization change was measured when light was sent through the TGG crystal once, and the change increased to 31.9 degrees when the light was sent back through the crystal. Experiments were also conducted to measure the Faraday effect caused by a piece of magnetically-permeable material, and to measure the magnetic field produced by a SmCo ring magnet in order to optimize the placement of the magnet. Using the TGG crystal, a distinct signal was generated every half-cycle of rotation, with an observed modulation depth of virtually 100 percent. Although data for a YIG based tachometer has not been recorded, the signal has been seen on an oscilloscope. The signal produced by the YIG-based device appears to be less intense than when TGG was used, but the signal to noise ratio can be made high enough to compensate for this difference. The thinness of the YIG sample (less than one millimeter) makes it more desirable than the TGG crystal, because it allows for a more compact design and makes it easier to induce Faraday rotation by modulating an applied magnetic field.

# 1 Introduction

Although it has been known for over 150 years that magnetic fields can influence light, only recently has the field of magneto-optics played an important role in numerous scientific and technological applications. In particular the Faraday effect, which is perhaps the most famous of the various magneto-optical effects, has been vital to the development of contemporary optical technologies.

This research grew out of a desire to study the Faraday effect in the context of a particular application: a magneto-optical tachometer. This device measures the rotational speed of a wheel by means of magneto-optics instead of electrical components. Optical sensors have a variety of advantages over classical electrical sensors. By the use of fiber optics, optical sensors can be made very small and thus very cheap and non-intrusive. They are also insensitive to electromagnetic interference, unlike sensors with conducting materials, and only require power for a source of light – most likely a compact, efficient laser. In recent years there has been a great interest in the design of optical current sensors, but relatively little attention has been paid to the design of optical tachometers.

In the Faraday effect, the plane of polarization of light is rotated when the light is passed through certain materials in the presence of a longitudinal magnetic field. Thus, if a magnetic field can be periodically modulated with the turning of a wheel, the polarization of light can likewise be affected. (Zook, Pollock 1993) Sensing this periodic fluctuation in polarization can reveal the frequency of the wheel's rotation, and so the Faraday effect can be used to construct a tachometer.

Instead of strapping a magnet to a wheel, the design used a more non-invasive approach. A magnetically-permeable rod mounted on a rotating shaft periodically modulated an existing field produced by a stationary permanent magnet. The effect on the light's polarization was slight, yet measurable. This design makes it unnecessary to firmly attach a magnet to a wheel shaft. And while the sensor that was built occupies an optical breadboard worth of space (18 x 36 inches), it could easily be made much smaller by the use of fiber optics.

## 2 Faraday Effect

In 1845 Michael Faraday discovered that applying a magnetic field to certain materials rotates the plane of polarization of light passing through the material. This magneto-optical phenomenon, later named in his honor as the Faraday effect, is proportional to the component of the magnetic field in the direction of the light's propagation ( $B$ ), and to the length of the material ( $L$ ). The proportionality constant  $V$  (called the Verdet constant) depends on the material, the temperature, and the frequency or wavelength of the light. Thus the amount of polarization change, measured as the angle of rotation, is given by  $\theta = BLV$ .

For magnetic crystals such as iron garnets,  $B$  is the magnetization of the material. As a stronger external field is applied, the magnetization will increase to saturation where there will be no more effect on the light's polarization. Saturation curves are close to linear when the applied magnetic fields are low, however.

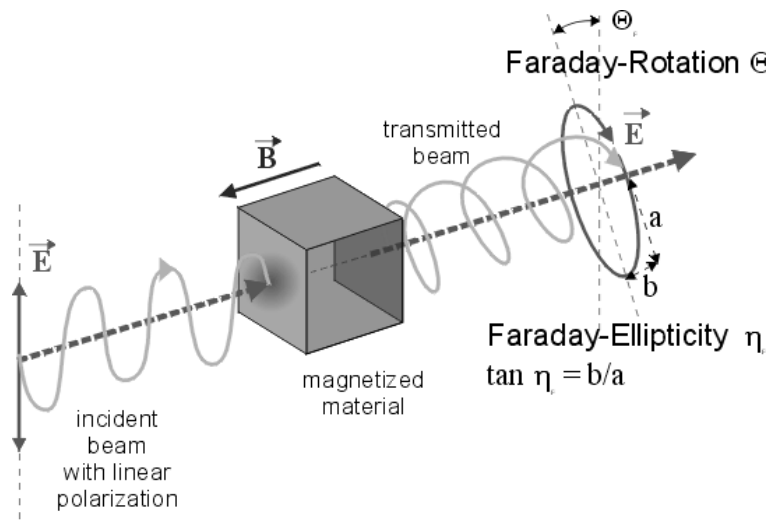


Figure 1: In the Faraday effect, the polarization of light is rotated when it is passed through certain materials in the presence of a magnetic field. (image from <http://www.physik.fu-berlin.de/ag-fumagalli/snom/#whatfaraday>)

The cause of the Faraday effect is complicated and a complete physical explanation cannot be made without an understanding of the quantum behavior of light and molecules. A mechanical model can help visualize the dynamics which lead to the Faraday effect, however. As polarized light passes through a medium, electrons are excited and oscillate parallel to the light's  $E$ -field. The electrons then re-emit light with a plane of polarization parallel to

their oscillation, and this continues throughout the medium. If a constant  $B$ -field is applied to the material in the direction of the light's propagation, a force  $v \times B$  will be experienced by the electrons perpendicular to both the  $B$ -field and their initial direction of oscillation. (Fowles 191) The direction of the electrons' oscillation will thus be rotated in the direction of the force, and the electrons will re-emit light whose direction of polarization is rotated similarly.

For demonstrations or applications of the Faraday effect it is desirable to achieve a significant change in polarization angle. This can be achieved either by using very long pieces of material with a low Verdet constant (water, glass), or by finding materials with high Verdet constants. A variety of rare-earth garnet crystals have extremely high Verdet constants, up to thousands of degrees per T cm. For this experiment, two of these rare-earth crystals – Terbium Gallium Garnet (TGG) and Yttrium Iron Garnet (YIG) – were used. For red light of 630 nm, the Verdet constant of TGG is roughly 77 degrees  $T^{-1}cm^{-1}$ , and is about twelve times larger for YIG. By comparison, the Verdet constant of water at that wavelength is less than 2 deg  $T^{-1}cm^{-1}$ , and for air it is only  $1.6 \times 10^{-3}$ . (Hecht 362)

Another interesting and useful property of the Faraday effect is that it is non-reciprocal, i.e., the rotation does not depend on the direction of the light's propagation relative to the direction of the magnetic field. Thus, placing a mirror at one end of a crystal to direct the light back through it will double the effect instead of canceling it out.

The Faraday effect has a variety of applications in addition to the tachometer idea. Certainly a Faraday effect-based sensor can be built to measure virtually anything relating to magnetic fields or changing electric fields, including electrical currents. In fact, there is a large market for non-electrical current sensors for high voltage currents. The Faraday effect can be used to study mixtures of hydrocarbons, since each molecule creates its own characteristic Faraday rotation. (Hecht 363) Recently the engineering department at UConn at Storrs has built a glucose-concentration sensor that exploits the Faraday effect of water and the natural polarization change produced by glucose. A magnetic field is applied to the solution through which light is passing until the change in polarization produced by the Faraday effect of water cancels the rotation caused by the glucose, and from this the glucose concentration can be calculated. (Nelson) One famous and extremely important application of the Faraday effect is the Faraday isolator. Faraday isolators utilize the non-reciprocity

of the Faraday effect to transmit light in one direction but not in the opposite direction. (EOTech) These optical elements are crucial for the elimination of reflections that can cause undesired optical feedback in diode lasers, for example.

### 3 Preliminary Experiments

Before the tachometer was built, a variety of preliminary experiments were conducted to test the materials and maximize the efficiency of the device. The apparatus and procedures common to all of the experiments are summarized in Appendix A.

#### 3.1 Demonstration of One-Directional Faraday Rotation in TGG

To demonstrate the Faraday effect, a simple setup was constructed to measure the amount of Faraday rotation when the light was passed through the TGG sample once. (fig. 2)

Polarized light from the diode laser was first sent through a lens of focal length 40 cm placed a similar distance from the TGG crystal. The lens produced a compact beam waist over the length of the crystal, which allowed the entire beam to pass through it. After exiting the crystal, the light passed through a rotatable polarizer and then to a photodiode where its intensity was measured. Intensity measurements were taken for every five degrees of the polarizer, with and without an applied magnetic field.

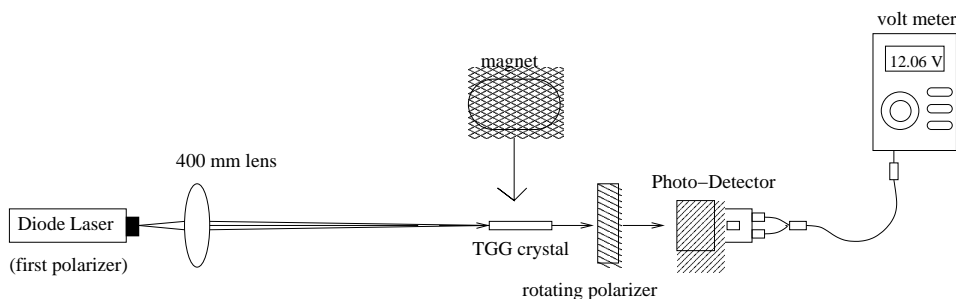


Figure 2: Setup for measurement of one-directional Faraday rotation in TGG.

When polarized light passes through a polarizer, only a component of its  $E$ -field is transmitted. In theory, if the polarizer and the polarization of the light are lined up, there will be 100 percent transmittance (in reality, most polarizers have a maximum transmittance of about 50%), and if they are oriented  $90^\circ$  to one another, no light will be transmitted.

More specifically, the component of the  $E$ -field that is transmitted is equal to the maximum  $E$ -field times the cosine of the angle of orientation between the polarization of light and the polarizer. Since light intensity is proportional to the square of the electric field, the light intensity is thus governed by the formula:

$$I(\theta) = I_0 \cos^2(\theta) \quad (1)$$

where  $I_0$  is the maximum intensity of the transmitted light and  $\theta$  is the angle of orientation between the light polarization and the polarizer. This is called Malus's law. (Hecht 326)

When there was no applied magnetic field, this is exactly what happened. A near perfect fit was made to the data by a cosine-squared curve. The data was fit by the equation:

$$I(\theta) = I_0 \cos^2(\theta + \Delta) \quad (2)$$

where  $\theta$  is the angle of the polarizer and  $\Delta$  is a constant that depends on the direction of the light's initial polarization.

When the magnetic field was applied to the crystal, the polarization of light was rotated by an angle  $\phi$  before entering the polarizer, and thus the angle of orientation was always increased by that same amount which became a phase shift. The data was thus fit by the equation:

$$I(\theta) = I_0 \cos^2(\theta + \Delta + \phi) \quad (3)$$

where  $\phi$  was the phase shift and was measured to be 15.1 degrees. Therefore, the TGG crystal rotated the polarization of the light by 15.1° in the presence of a magnetic field.

Since the dimensions of the crystal are known, all that was needed to calculate the Verdet constant was the strength of the applied magnetic field. In this experiment, a strong permanent magnet was used. Since it is known that this magnet is non-uniform, the calculation was difficult. When the applied magnetic field  $B$  is non uniform, the term  $LB$  in the equation  $\theta = BLV$  becomes the integral of the magnetic field along the distance  $L$ :

$$\theta = V \int B dL \quad (4)$$

For a non-uniform magnet the integrated field can be very complicated and even unpredictable. Using a gauss-meter, it was found that along the length of the crystal the magnetic field intensity rose and then fell, with a maximum at the center. While precise measurements were not taken, the field was about 1200 gauss at the ends of the crystal and 1800 gauss at

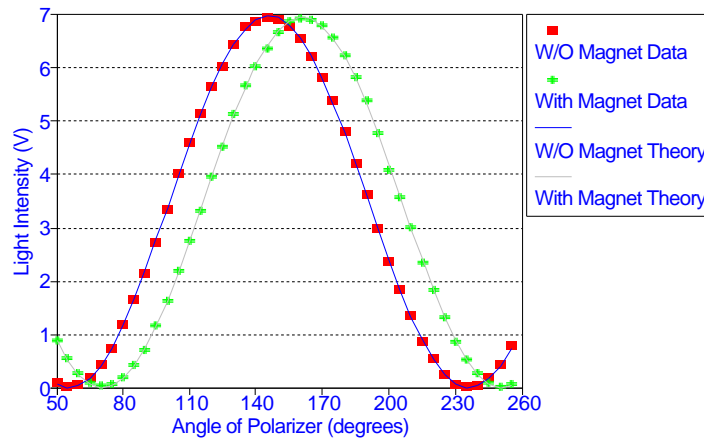


Figure 3: Light intensity *versus* polarizer angle with and without an applied magnetic field. The distinct  $15.1^\circ$  phase shift is caused by the Faraday rotation.

the maximum. Thus, the average field intensity was probably somewhere between 1400 and 1600 gauss. Using 1500 gauss, or .15 T as an estimate, the Verdet constant was calculated to be  $77.6 \text{ deg T}^{-1}\text{cm}^{-1}$ . The published Verdet constant for TGG is  $76.7 \text{ deg T}^{-1} \text{ cm}^{-1}$  at 630 nm. (Moltech)

### 3.2 Demonstration of 2-Directional Faraday Rotation in TGG

The next experiment was designed to demonstrate the non-reciprocal Faraday effect when the light is sent back through the crystal. (See Figure 4.)

Light from the diode laser was sent through the lens, polarizer and crystal as before, except that a beam-splitter was now placed between the lens and the crystal. The beam-splitter passed approximately one-half of the light unchanged and diverted the other (wasted) half to a beam catcher. After exiting the crystal, the undeviated light was reflected by a mirror back through the crystal. It then passed through the polarizer again, which acted as an analyzer. Once again, about one-half of the returning light was deflected at  $90^\circ$ , where it entered the photodetector, and the other one-half was wasted.

The light intensity was again measured in  $5^\circ$  steps of the polarizer, with and without an applied magnetic field. An observable phase shift of  $30.2^\circ$  (twice the previous experiment) was anticipated.



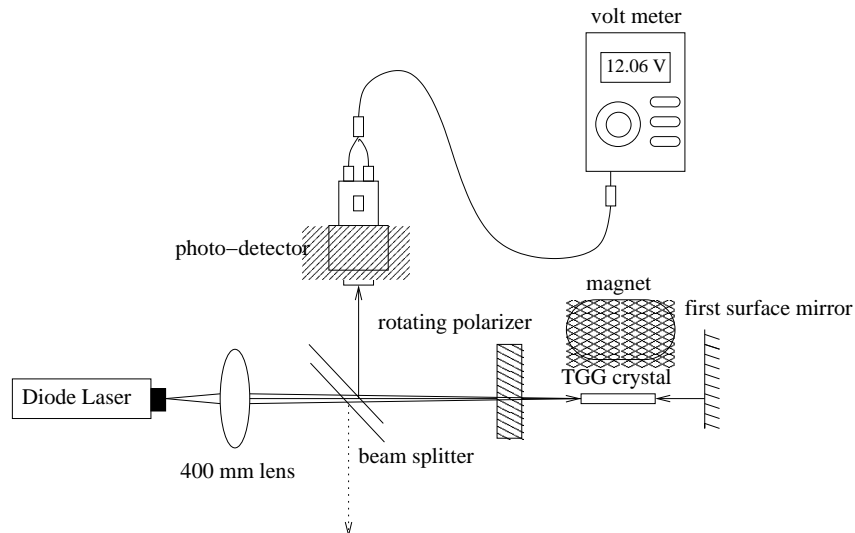


Figure 4: Setup for measurement of Faraday rotation when the light was sent through the crystal twice.

Instead of producing two nice cosine-squared curves with a large phase shift, there was *no* phase shift, there was a constant amplitude ratio, and the graphs were not quite cosine-squared. The asymmetry of the graphs is probably a result of the use of a beam-splitter. Reflection and transmission through a dielectric such as a beam-splitter can partially polarize light, and this will affect experiments in which a polarizer is used (see Appendix B).

After some thought and discussion it was realized that the non-existent phase shift and the constant amplitude ratio was because of the placement of the polarizer. Consider polarized light emitted from the diode laser that passes through the polarizer. The intensity of this light after it exits the polarizer will be:

$$I(\theta) = I_0 \cos^2(\theta + \Delta) \quad (5)$$

where  $\theta$  is the angle of the polarizer and  $\Delta$  is a constant that depends on the laser orientation (the initial polarization of the light). The polarization of this light will now be parallel to the polarizer. After passing through the crystal twice in the presence of a magnetic field, the polarization will rotate by an angle  $2\phi$ . Thus, when the light re-enters the polarizer, its polarization will always be oriented an angle  $2\phi$  with the polarizer. Thus the intensity of the light will be scaled by a factor  $\cos^2(2\phi)$ , and the overall intensity will equal:

$$I(\theta) = I_0 \cos^2(2\phi) \cos^2(\theta + \Delta) \quad (6)$$

where  $\phi$  is a constant that is the one-directional Faraday rotation.

Since when there is no magnetic field the light intensity varies as  $I_0 \cos^2(\theta + \Delta)$ , the constant ratio between the two graphs is simply  $\cos^2(2\phi)$ . To calculate the two-directional Faraday rotation seen in this experiment, this constant ratio was found by averaging the ratio of the data points. The ratio was about 0.72, which implies a two-directional Faraday rotation of  $31.9^\circ$ , in good agreement with the predicted  $30.2^\circ$  rotation.

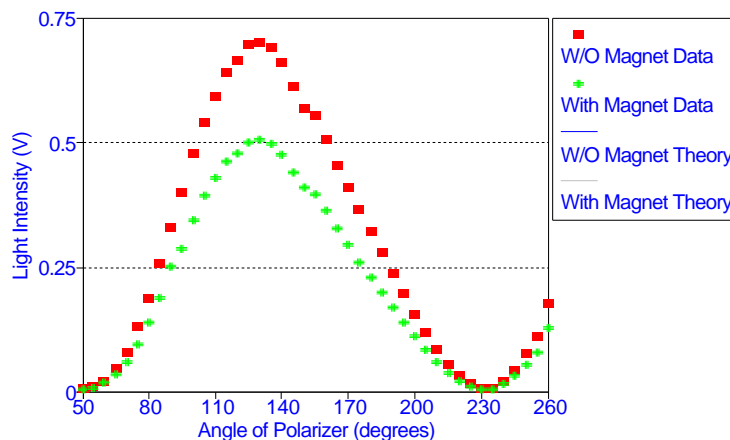


Figure 5: Light intensity *vs.* the angle of the polarizer when the laser light was sent back through the crystal, with and without the presence of a magnetic field. The asymmetrical shape and lack of a phase shift were explained by the use of a beam-splitter and the placement of the polarizer.

For the construction of the tachometer, it was decided that the light should pass through the crystal twice to double the Faraday rotation, but only once through the polarizer to produce a phase shift similar to the first experiment but twice as large.

### 3.3 Modulating the Magnetic Field to Produce Faraday Effect

Since the Faraday effect is proportional to the longitudinal component of the magnetic field strength, any change in this component will result in a change in polarization, and even a slight fluctuation can be detected. To make the tachometer more practical, it was designed to respond to slight changes in magnetic field intensity brought about by the presence of a magnetically permeable rod. Since, in general, it would probably be easier to attach a metal rod to a shaft than to attach a magnet, this design makes the tachometer more versatile and

less intrusive. An experiment was designed to measure the effect of a piece of magnetically permeable material on the amount of polarization change.

It is important to note that from this point in the project on, the magnet used was the Samarium Cobalt (SmCo) ring magnet described in Appendix A. The field produced by this extended ring magnet was more predictable (see Appendix C).

To examine the effect of the magnetically-permeable material (a steel screw) on the Faraday rotation, the setup was the same as for the measurement of the two-directional Faraday rotation, except that the crystal was inserted inside the ring magnet, and the polarizer was placed in front of the photodiode so the light would only pass through it once. Intensity readings were recorded for every five degrees of the polarizer with and without the presence of the steel screw. The presence of the steel screw rotated the light's polarization by about  $0.7^\circ$ , so there were certain angles at which the intensity change was greatest. To find the angle at which the greatest change occurred, the intensity differences were plotted. (Figure 6.) In addition, fits were made to the intensity graphs with and without the presence of the screw, and the difference between these two fits were plotted. It was found that the maximum intensity difference occurred when the polarizer was oriented at  $108^\circ$ .

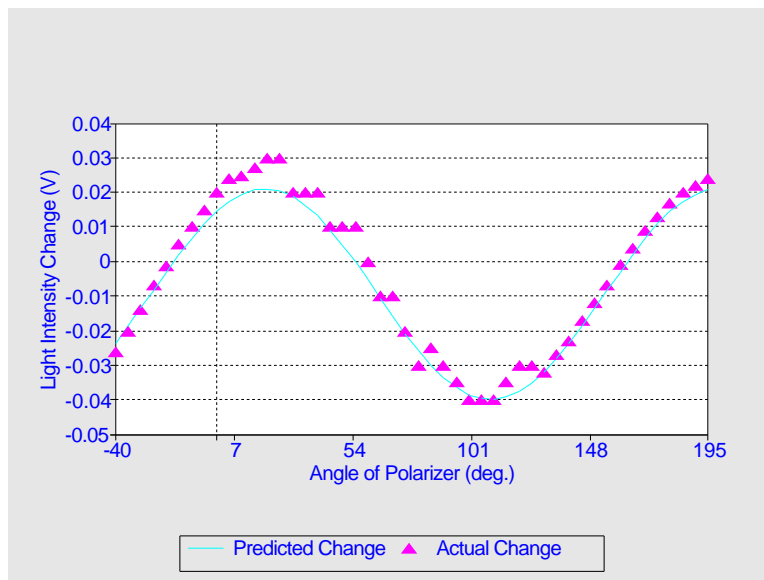


Figure 6: When a magnetically permeable steel screw was placed near the magnet and the crystal, the intensity of the light changed. The maximum intensity change occurred when the polarizer was set to  $108^\circ$ .

## 4 Construction of the Tachometer

The tachometer was set up like the experiment for measuring the Faraday effect in the presence of the steel screw, except that the screw was replaced by a stepping motor with a mu-metal cylinder mounted on the shaft. Mu-metal is a material with an extremely high magnetic permeability (symbol  $\mu$ ). As the stepping motor rotated the mu-metal, it modulated the magnetic field produced by the stationary ring magnet, causing the light's polarization to fluctuate slightly, thus changing the light intensity every half-cycle.

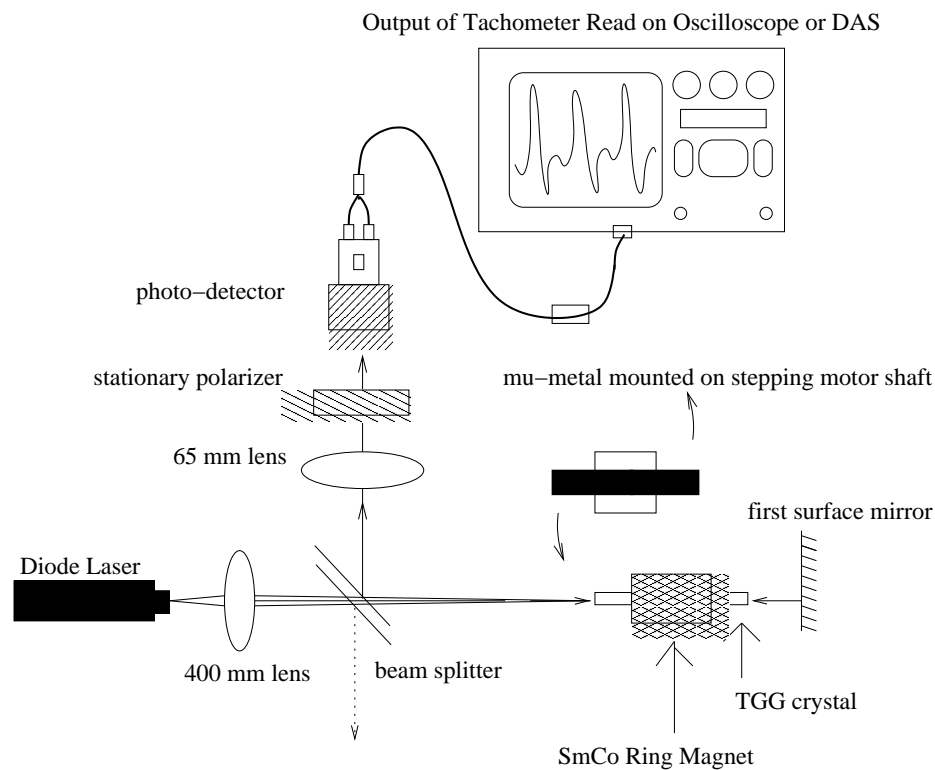


Figure 7: Setup for the tachometer. As the shaft rotated the mu-metal, the magnetic field intensity periodically fluctuated inside the crystal, changing the polarization and thus the intensity of the laser light every half cycle.

## 5 Results

The tachometer was tested at two different frequencies - slightly more than 2 Hz, and about 5 Hz. Light intensity was recorded by a Data Acquisition System (DAS) every millisecond for one second. The cable from the photodiode to the DAS was AC coupled with a capacitor to only show changes in voltage, which is the relevant quantity. The two trials show distinct intensity peaks every half-cycle of rotation, with a decent signal to noise ratio.

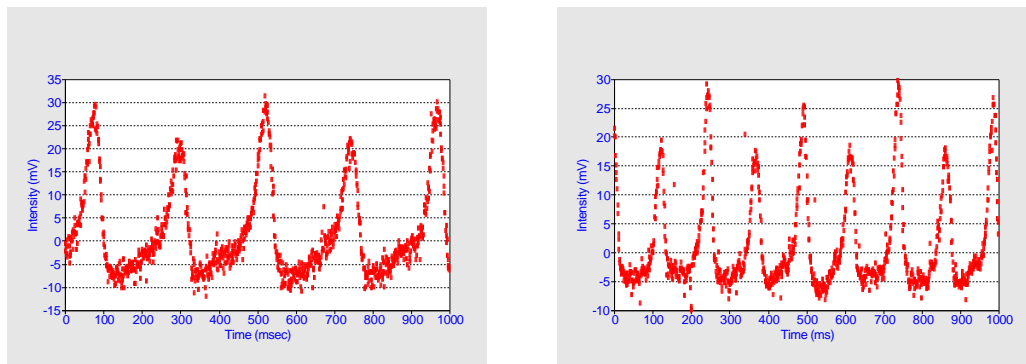


Figure 8: Output of the tachometer versus time. Notice the difference in modulation depth between any two adjacent peaks. This is because the mu-metal was mounted off center on the shaft, making the distance between the metal and the magnet slightly different every half cycle. The revolution period in the first trial (left), is just under 500 ms (frequency  $f = 2$  Hz); in the second trial (right), the speed of the motor was increased to about 5 Hz.

## 6 Future Experiments

Experimentation has already begun with a tachometer based on a YIG crystal instead of the TGG crystal. Using an oscilloscope, a similar modulation depth percentage was observed, but the signal was slightly weaker. This is still significant, because the signal to noise ratio can be increased by rotating the polarizer, so the actual size of the light intensity change is not very important. The YIG crystal, which was sent from Osnabruek University in Germany, is more favorable than the TGG crystal because of its tiny size. Grown on a microscope slide, the sample of YIG is less than one millimeter thick. Not only is it advantageous to have a smaller crystal because it can reduce the sensor size, but a smaller crystal makes it much

easier to induce a rotation from a modulated  $B$ -field. Since Faraday rotation is proportional to the integral of the magnetic field along the crystal, simply changing the magnetic field at one point inside the 1.3 cm long TGG crystal would not induce a significant change in polarization, and attention must be paid to the  $B$ -field along the entire length of the crystal. The thickness of the YIG allows it to be treated like a point, however, and it is much easier to manipulate the field to produce a large change at a single point than for over a centimeter.

Using both the TGG and the YIG crystals I would like to gather more data and be able to compare the results over a wider range of frequencies. I want to know if there is an upper limit to the frequency that can be read by this device, and if there is, how to design a tachometer with a wider range. I would also like to redesign several of the components used in the setup to be more stable. The vibration of the laser due to an unstable mounting design was the primary source of noise, so a more stable mount would lead to a much better signal-to-noise ratio. The motor itself should also be held down more firmly to reduce vibrations. In addition, if I could reduce the size of the slide on which the YIG sample was grown so that the crystal could fit inside the ring magnet, the output of the tachometer would be much more pronounced. To increase the modulation depth of the signal I could also use a stronger magnet, or use a thicker piece of mu-metal.

In the future it will also be possible to reduce the size of the tachometer by means of fiber optics. The sensor head – including the crystal, the magnet, and a polarizer – could be very small and completely self contained. Polarized diode laser light could be sent through an input fiber, and output through a second fiber in the same direction. Thus the laser and the photodiode, the two space and power consuming components of the device, would be completely isolated from the sensor head itself. With no lead-in electrical cables, the sensor could go virtually anywhere non-intrusively. The construction of such a device is the ultimate goal of this research.

## 7 Conclusion

The magneto-optical design for a tachometer based on the Faraday effect works well with standard optical equipment and TGG or YIG crystals. The near 100 percent modulation depth with a good signal to noise ratio was more than adequate for frequency measurements.

There are a number of advantages to using this optical device, including the potential to have a compact design, no lead-in electrical cables, and insensitivity to interference. The only power needed to operate the tachometer would be enough to run a 5 mW (or lower) laser, which will work for days on two AAA-batteries. There is definite potential for this design to serve as a backup system to current tachometers that would protect against power failure – especially in aircraft or other machines vulnerable to power loss, in which a working tachometer is constantly needed.

In fact, the sensor may be even more versatile, because it could plausibly be used to measure things other than the rotational speed of a wheel. The sensor could detect the presence of magnetically permeable material nearby, for example, and could respond to very slight changes in magnetic field intensity. Because the sensor does not require direct measurements and calibration, it may be much easier to detect the presence of a magnetic field with this sensor rather than a gauss probe, as long as an exact measurement isn't needed. And of course, there may be other uses for this device that have not been thought of yet; only future investigation will tell.

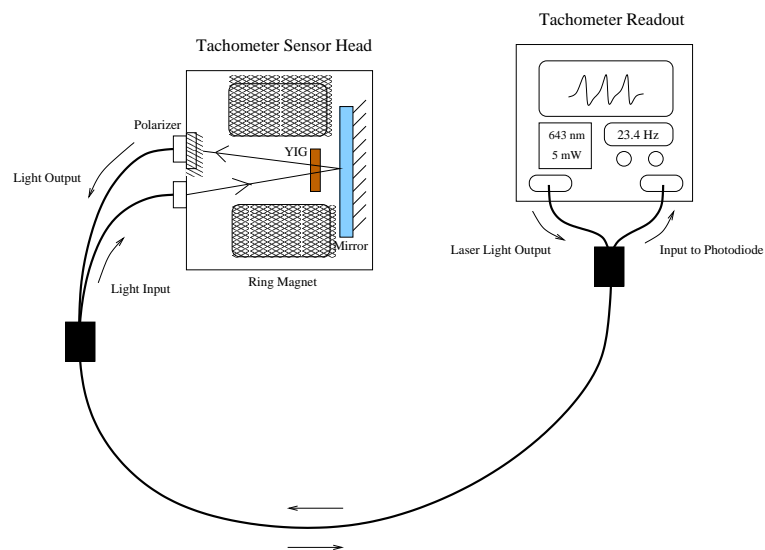


Figure 9: The ultimate goal is to build a fiber optic tachometer with a very small sensor head. The laser input and the output to the photodiode would be completely separated from the sensor head, connected only by two small optical fibers.

## Acknowledgements

I would like to thank Prof. Harold Metcalf and Dr. John Noé of the Laser Teaching Center at SUNY Stony Brook for devoting so much time, help, and insight to my project. Special thanks to Tina Shih for all her help around the lab, and to Osnabruek University for donating the YIG sample. This research was made possible by the Simons Fellowship program.

## Resources

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## Appendix A: Apparatus and Procedures

The experimental setups were all assembled on a small ( $18 \times 36$  inch) optical breadboard using standard components and mounts from Thorlabs.

- \* The diode laser was a typical pen-sized  $<5$  mW laser pointer powered by two AAA batteries. The on-off button was held down with a tie wrap, and the laser was turned on or off by unscrewing the end cap. With the help of a small adapter piece the laser was epoxied into a Thorlabs RP-1 rotating mount, which was convenient for orienting the plane of polarization.
- \* The rotating polarizers were made by gluing pieces of Edmund Scientific brown Polaroid sheets on to Thorlabs RP-1 rotating mounts. The rotating mounts were accurate to  $1^\circ$ .
- \* The photodetector was a Thorlabs DET-110 with a  $100\text{ k}\Omega$  resistor across the output to convert current to a voltage in the range  $0 - 10$  volts.
- \* Two types of permanent magnets were used. One was irregularly shaped and of unknown material. For tachometer measurements, 4 SmCo ring magnets (Edmund Scientific #R31-571) held together by their mutual attraction were used.  $B$  field measurements were made using a longitudinal Hall effect gauss-meter. Micrometer driven translation stages were used for the measurements of magnetic field *versus* position.
- \* The rotating mu-metal cylinder used for the tachometer was a surplus magnetic shield for a small photomultiplier tube. The 10 cm long by 3 cm diameter cylinder is made from rolled mu-metal sheet 0.25 mm thick. The stepping motor was an M061 type operated from a power supply box driven by a pulse generator (200 pulses/s).
- \* The Terbium Gallium Garnet (TGG) crystal sample used has the shape of a small cylinder, 4.8 mm in diameter by 13 mm long. The two circular ends are polished, but also chipped around the edges, limiting the best light transmission to the center portion. Other experiments were done with a thin sample ( $<1$  mm thick) of Yttrium Iron Garnet (YIG) on loan from Osnabruek University in Germany.
- \* Data analysis and plotting was done with the QuattroPro for DOS spreadsheet on a 486/33 computer. Plot files were saved in .eps format and transferred to a linux computer for printing or use in latex documents such as this report. The fluctuating voltages produced by the tachometer were read and recorded with the Keithley DAS-1600 data acquisition system and plotted in the same way.

## Appendix B: Reflection at a Dielectric Surface

In the experiment that measured the two-directional Faraday rotation of the TGG crystal, it was found that the beam-splitter altered the polarization of the laser light. This is a result of the dielectric nature of the beam-splitter. The laser light can be thought of as the superposition of two plane polarized beams – one parallel to the reflective surface (*p*-polarization) and the other perpendicular to the surface (*s*-polarization).

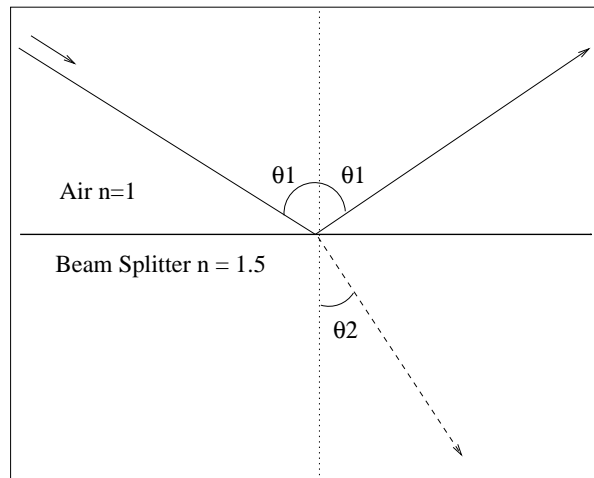


Figure 10: When light is incident on a dielectric boundary, some is reflected and some is transmitted. The angle of incidence  $\theta_1$  (which is equal to the angle of reflection) is related to the angle of refraction  $\theta_2$  by Snell's law  $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ , where  $n_1$  and  $n_2$  are the indices of refraction of the two media.

The reflectivity is different for the *s* and *p* components of polarization at a dielectric surface, and both depend on the angles of incidence and refraction according to Fresnel's laws (Melles Griot 4-5). If  $\theta_1$  and  $\theta_2$  are the angles of incidence and refraction, then the reflectivity ( $r$ ) of the *s* and *p* components of the light's polarization is given by:

$$r_s = \left[ \frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)} \right]^2 \quad (7)$$

$$r_p = \left[ \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)} \right]^2 \quad (8)$$

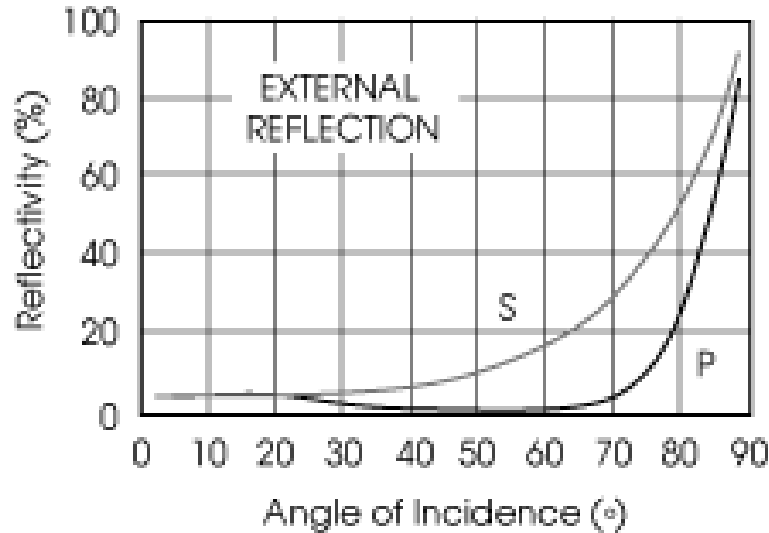


Figure 11: The *s* and *p* components of polarization have different reflectivities which depend on the angle of incidence  $\theta_1$ . At Brewster's angle, here about  $57^\circ$ , no *p*-polarized light is reflected.

(image from <http://optics.org/cvi/appendix/refltranphase.html> )

There is always less reflectance of the *p*-polarized light beam than the *s*-polarized beam, so light reflected off a dielectric surface is partially polarized. In fact, according to Fresnel's equations there is an angle of incidence  $\theta_B$  at which *no* *p*-polarized light is reflected – the reflection is composed of only *s*-polarized light and is therefore completely polarized. This angle is called Brewster's angle, and occurs when  $\theta_1 + \theta_2 = 90^\circ$  (Melles Griot 4-6). This can also be written as:

$$\theta_B = \arctan\left(\frac{n_1}{n_2}\right) \quad (9)$$

For glass (or a beam-splitter) with an index of refraction of  $n \simeq 1.5$ , Brewster's angle is about  $57^\circ$ . In the experiment, light reflected off the beam-splitter at  $45^\circ$ , which is close enough to Brewster's angle to have a significant effect on the polarization.

The phenomenon of Brewster's angle can be demonstrated by viewing light reflected from ceramic floor tiles through a polarizer. As the polarizer is rotated, the reflected light will alternately dim and brighten because it is partially polarized. The effect is more pronounced near Brewster's angle.

## Appendix C: The B-Field of the SmCo Ring Magnet

The samarium cobalt ring magnet was used because its magnetic field was more predictable, thus making it easier to manipulate with the rotating piece of mu-metal. To help understand its magnetic field, a model was made to represent the horizontal component of its  $B$ -field along the  $z$ -axis which runs through the center of the ring. From informal experiments with a gauss probe, it was hypothesized that the ring magnets behaved like a bar magnet rotated about the  $z$ -axis. Thus the field lines were expected to wrap around each part of the magnet, from face to face, on the outside and the inside, as illustrated in figure 12.

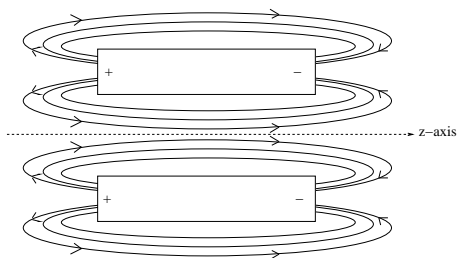


Figure 12: A cross section of the ring magnet shows how each lengthwise segment acts like its own bar-magnet, creating field lines that loop around from face to face on the inside and outside of the magnet.

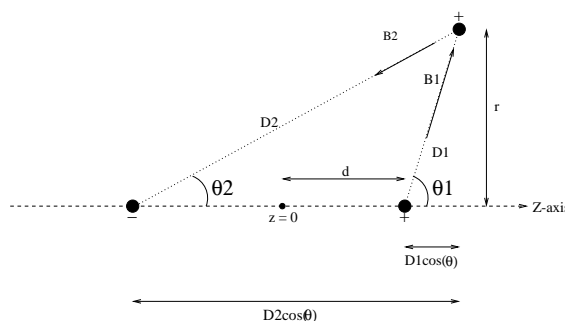


Figure 13: The  $B$  field along an axis parallel to the  $z$ -axis was modeled by calculating the field created by a dipole at this distance.

Assuming that the field behaves in this manner, it is possible to calculate the horizontal magnetic field strength along an axis parallel to the  $z$ -axis as a function of  $z$  by using an electric dipole model. Assume that the magnet consists of two point charges, one positive and one negative, located a distance  $d$  from either side of the  $z$ -axis origin (figure 13).

To find the field along a parallel axis a distance  $r$  from the  $z$ -axis, the field is calculated at a point  $(z, r)$ , with  $r$  being constant and  $z$  being a variable. As shown in figure 13, the magnitude of the field consists of a contribution from the positive and negative charges. Consider the contribution from the positive charge. The field varies as the inverse square of the distance from the charge,  $D_1$ , so (assuming a scalar amplitude will be included later to represent the strength of the poles) we can write:

$$B_1 = \frac{1}{D_1^2} = \frac{1}{(\sqrt{(z-r)^2 + r^2})^2} = \frac{1}{(z-r)^2 + r^2} \quad (10)$$

Since we are only concerned with the  $z$ -component, we must now multiply the magnitude of  $B_1$  by  $\cos(\theta_1)$ . Here,  $\cos(\theta_1)$  can also be written as  $(z - r)/\sqrt{(z - r)^2 + r^2}$ , so the  $z$  component of the  $B$ -field becomes:

$$B_{1z} = B_1 \cos(\theta_1) = \left[ \frac{1}{(z - r)^2 + r^2} \right] \left[ \frac{z - r}{\sqrt{(z - r)^2 + r^2}} \right] = \frac{z - r}{[(z - r)^2 + r^2]^{\frac{3}{2}}} \quad (11)$$

The contribution from the negative charge will be similar, but its sign will be negative and the term  $z - r$  will always be replaced with the term  $z + r$ . Thus the model for the axial magnetic field strength through the center of the ring magnet is:

$$B(z) = B_0 \left[ \frac{z - r}{[(z - r)^2 + d^2]^{\frac{3}{2}}} - \frac{z + r}{[(z + r)^2 + d^2]^{\frac{3}{2}}} \right] \quad (12)$$

where  $B_0$  is a constant that depends on the strength of the magnet.

Using the dimensions of the magnet ( $r = 0.64$  cm,  $d = 0.740$  cm), a model of the field strength throughout the magnet was calculated. Then using a gauss-probe, the actual values of the field strength along the center of the magnet were recorded and plotted. The theoretical calculation makes an excellent fit to the data.

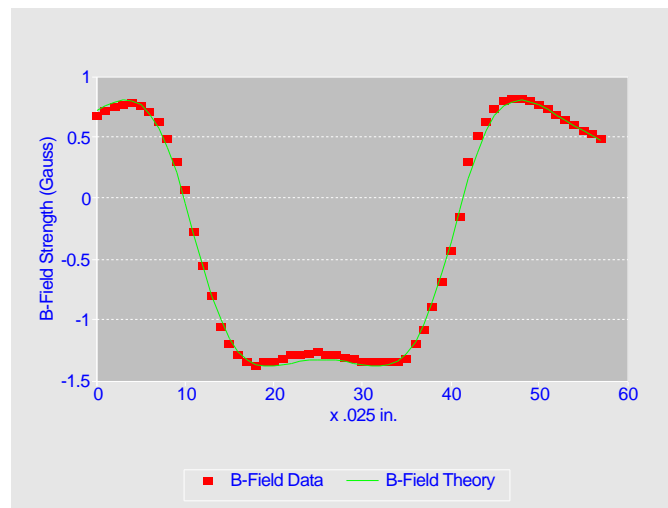


Figure 14: Plotted here is axial magnetic field strength versus distance inside the magnet: both a theoretical model and experimental values. The theoretical model was successful, even predicting the small “bump” in field strength at the center of the magnet.